UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE ECONOMÍA

TESIS

INTERTEMPORAL COMPETITIVE EQUILIBRIUM: A COMPUTATIONAL MODEL OF INTERMEDIACY IN SPECULATIVE MARKETS

POR
JOSÉ JAVIER GONZÁLEZ GUTIÉRREZ

PRESENTA COMO REQUISITO PARCIAL PARA OBTENER EL GRADO DE DOCTOR EN ECONOMÍA

AGOSTO, 2015

UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE ECONOMÍA
DIVISIÓN DE ESTUDIOS DE POSGRADO

INTERTEMPORAL COMPETITIVE EQUILIBRIUM: A COMPUTATIONAL MODEL OF INTERMEDIACY IN SPECULATIVE MARKETS

POR
JOSÉ JAVIER GONZÁLEZ GUTIÉRREZ

TESIS PRESENTADA COMO REQUISITO PARCIAL PARA OBTENER EL GRADO DE DOCTOR EN ECONOMÍA

AGOSTO, 2015

“INTERTEMPORAL COMPETITIVE EQUILIBRIUM: A COMPUTATIONAL MODEL OF INTERMEDIACY IN SPECULATIVE MARKETS”

J Javier González G

Aprobación de Tesis:

Asesor de la Tesis **DR. LORENZO BLANCO GONZÁLEZ**

DR. LUIS HUESCA REYNOSO

DR. ERNESTO AGUAYO TÉLLEZ

DR. MARCO VINICIO GÓMEZ MEZA

DR. IGNACIO LLAMAS HUITRÓN

DR. ERNESTO AGUAYO TÉLLEZ
Director de la División de Estudios de Posgrado
De la Facultad de Economía, UANL
Septiembre, 2014.

Intertemporal Competitive Equilibrium: A computational model of intermediacy in speculative markets

July 30, 2015

Introduction

> "For thirty-six million citizens to go and fetch the corn they want from Odessa, is a manifest impossibility.... The consumers cannot act for themselves. They must, by necessity, have recourse to intermediates...." -Frédéric Bastiat, "The Intermediates"

How are prices set in a market economy with decentralized information? How are we able to extract full gains from trade in a competitive market without centrally coordinated price formation and resource allocation? In 1962, separate research efforts proved that neither *rationality* nor *complete information* are necessary market conditions to reach competitive equilibrium[1]. Behavioral extensions to this framework have shown significant progress towards a coherent model of competitive price formation under varying conditions of decentralized trade and individual rationality. Unfortunately, economics continues to lack a generally valid model of decentralized price formation.

Advances in technology and methods has led to substantial testing of the institutional and behavioral boundaries of the standard microeconomic framework. Models using different learning behavior bestow *irrational* agents with varying cognitive abilities to reach competitive equilibrium. Similarly, models with different decentralized trading mechanisms show different rates of convergence to the equilibrium price as well as different levels of allocative efficiency (Smith, 1982; Plott, 1982; Gode, Sunder, 1997). Few efforts to modify the standard framework have been made to provide a model that proves to be consistent with competitive equilibrium outcomes under a broader set of market conditions. Using empirical evidence and analysis, they postulate that introducing speculative intermediacy into the microeconomic framework can allow models to capture a dynamic process of competitive price formation at larger scales. But according to the prevailing literature, an effort to synthesize these generalizations into a working framework is yet to be found.

Based on the works done by Becker (1962) and Smith (1962), Gode and Sunder (1993) show that exchange among *boundedly irrational*[2] agents in a non-tâtonnement trading institution can sustain high levels of allocative efficiency. Recent behavioral extensions use *adaptive learning*[3] models that provide market outcomes consistent with competitive equilibrium in stationary environments

[1] Working separately in 1962, Gary Becker and Vernon Smith were able to prove that neoclassical market-level predictions are consistent with a broader set of agent behavior and market institutions than omniscient rationality and Walrasian tâtonnement. Becker(1962) proved irrational agents with bounded opportunity sets where able to reach equilibrium outcomes through a tâtonnement trading process. Smith (1962) showed that a decentralized trading mechanism with rational agents can reach levels of efficiency similar to a tâtonnement trading process.

[2] A *bounded irrational* agent is cognitively limited, has no memory, and chooses randomly from a non-negative and bounded opportunity set – i.e., *no-loss condition.*

[3] Adaptive learning behavior describes the strategic making of decision based on experience

with low volumes of trade. (Crockett, Spear, Sunder, 2008; Duffy, Ünver, 2006).

Parting with neoclassical tradition, Spulber (1996) proposes incorporating intermediation into the standard economic framework to determine how markets reach equilibrium outcomes while adjusting to changes in supply and demand. In concept, the mediation of trade between buyers and sellers by *price-setting intermediaries* (dealers) motivated by profits allows markets to allocate resources efficiently. Rust and Hall (2003) present an extension to Spulber's model where buyers and sellers have the additional option of trading in an exchange operated by specialist (*market-makers*) with publicly posted prices. The evidence shows that adding a market-maker increases expected gains from trade for buyers and sellers while reducing *dealers'* profit-margin.

Substituting an impromptu matching exchange mechanism with intermediary spot markets fails to capture direct exchange among buyers and sellers and intertemporal market price formation. Miller et. al. (1977) addressed these issues in an empirical study of intertemporal competitive equilibrium in speculative markets. Just as complete information is not necessary a necessary condition in intratemporal markets, *perfect foreknowledge* of supply and demand is not a necessary precondition for a seasonal market to reach competitive equilibrium outcomes. Spulber (1998) develops a conceptual model of a multilateral exchange with publicly posted prices where buyers and sellers can trade directly with each other or trade with *market-makers*. Spulber suggests that market-makers competing to gain an arbitrage advantage drives transactions costs down and increases the welfare of consumers and producers.

This study is an attempt to synthesize these insights into a behavioral model of intermediation in speculative markets. I present an extension to Gode and Sunder (1993) multilateral exchange model with individual adaptive learning behavior (Duffy, Ünver, 2006). Buyers and sellers use *reinforcement learning*[4] to formulate bids and offers based on a convex combination of individual valuation and a prevalent market price subject to budgetary restrictions. Using concepts from Spulber (1998) I introduce a cohort of market-makers with specialized information and the capacity to hold cash and keep inventory across periods. The bidding behavior of the market-maker is a hybrid of reinforcement learning and *belief-based learning*[5]. Market-makers update their bidding and offering strategies based on a set of expected future market clearing prices and the current market average price. They are willing to provide additional

and observational history. In a comparative study by Feltovich (2000) adaptive learning models outperformed Nash equilibrium in asymmetric information experimental games. A survey of learning behavioral computational and experimental models can be found in Camerer (2003).

[4]This is the 'law of effect' (Thorndike, 1911). Actions or strategies that have yielded relatively higher payoffs in the past are more likely to be played in the future.

[5]In belief-based learning models, the player recognizes that he is playing a game and forms beliefs about the likely play of others (Duffy, 2006).

liquidity and immediacy in the market based on a perceived dynamic arbitrage profit according to their forecast of future market outcomes. This allows them to coordinate exchange among buyers and sellers across periods, which leads to increase in intertemporal allocation efficiency.

Related Literature

The neoclassical competitive market theory of supply and demand price determination postulates a *Walrasian tâtonnement* mechanism of resource allocation among 'utility-maximizers' endowed with complete albeit imperfect information. The underlying market process is regarded as a *central auctioneer* collecting bids and offers simultaneously while assigning a market price instantaneously. Under a construct of bilateral exchange with two goods, equilibrium analysis implies a market clearing price and efficient resource allocation as a consequence of individual optimization.

The notion of a market price somehow emerging from a groping process of exchange among omniscient optimizers, still prevalent in textbooks, no longer permeates the research literature. Becker (1962) proved basic features of economic theory such as a downward sloping demand curve can be derived in markets where *irrational* individuals have bounded feasibility sets. That is, competitive market outcomes can be achieved under random choice behavior subject to budget restrictions. In his seminal work on experimental markets with a multilateral exchange process, Smith (1962) concluded that a decentralized system of trade is able to reach equilibrium outcomes. According to the experimental evidence, inexperienced student traders with private valuations could extract 100% of the trade surplus in markets with publicly posted prices. Incidentally, Smith observed that an increase in the number of trade contracts led to a faster convergence towards equilibrium outcomes – even with a handful of students.

Over the past half century these findings have pervaded research efforts to explain the decentralized process of competitive price formation. The prevailing consensus in the experimental literature is that competitive models work best when markets are organized as *double auctions* (Plott, 1982). Research evidence shows that *double auction* markets converge to competitive equilibrium prices and full allocative efficiency with a few participants, regardless of the completeness of information and agent rationality (Friedman, 1984; Plott, 1982; Smith, 1982; Smith, Williams, Bratton, Vannoni, 1982). Additionally, the stationary repetition – i.e., static replication – of trading rounds of finite duration improves information aggregation, leading to faster convergence to competitive equilibrium and nearly full allocative efficiency (Friedman, 1984; Gode, Sunder, 1997; Miller, Plott, Smith, 1977; Plott, Sunder, 1988). In light of these findings, I choose the double auction trading mechanism and forgo any comparative institutional analysis to focus on gaining further insight into the behavioral process that leads markets towards competitive outcomes.

Based on the framework used by Smith (1962), Gode and Sunder (1993) set out to test the allocative efficiency of a *double auction* market with cognitively limited automated traders. By replacing humans subjects with *zero-intelligence* (ZI)[6] machine traders, they were able to show that high levels of allocative efficiency can be sustained without a tâtonnement trade mechanism or individual optimization. Additionally, ZI-traders subject to budget constraints – *no-loss restriction* – were able to reach allocative efficiency levels statistically indistinguishable from those of their human counterparts. Market discipline imposed on traders, not trader cognition, is the primary cause of the high allocative efficiency in a multilateral exchange market (Gode, Sunder, 1993).

Gode and Sunder (1993) modeling framework has become the platform for behavioral models of markets with a multilateral exchange process. Crockett et. al. (2008) built a computer microsimulated market to test the effects of added trader cognition on allocative efficiency in a zero-intelligence environment. They propose an algorithmic model as an alternative to Walrasian tâtonnement where boundedly rational agents with neoclassical preferences learn competitive equilibrium in a repeated static exchange economy. The simulated evidence shows the existence of an informationally decentralized institution and a set of behaviorally plausible strategies capable of reaching competitive equilibrium outcomes.

Extending on Gode and Sunder (1993) automata simulation, Duffy and Ünver (2004) built an *agent-based computational economic* model (ACE)[7] of a double auction market with cognitively superior traders. These *near-zero-intelligence* (NZI) traders formulate bids and offers as a convex combination of randomly generated prices and the mean traded price from the previous period. The simulated market outcomes are not only qualitatively similar to the experimental data but it many instances they closely approximate their magnitude in volume and price changes. Duffy and Ünver conclude that the *ZI* approach effectively exposes the significance of institutions and other economic environment features relative to human cognition in the determination of observed market outcomes.

These computational efforts have expanded the set of market economies known to be consistent with competitive equilibrium predictions. But they are unable to explain the decentralized process of competitive price formation in nonstationary markets with large fluctuations in trade volume. Crockett et. al.

[6]ZI-traders are computer simulated automata generating independent and identically distributed random bids and offers from a set of trading prices, without memory or profit maximization motive.

[7]The use of agent-based simulations in economics have been categorized as Agent Based Computational Economic Models (ACE). ACE models simulate autonomous agents interacting in an environment given a set of rules to assess the performance of a system as a whole.

(2008) credit the inability of their model to coordinate trade with more than two traders due to the lack of an intermediary institution. They posit the use of an intermediary as a way to fully extract gains from trade. Duffy and Ünver (2004) acknowledge their model's inability to capture how increased experience affects trading behavior in laboratory markets. This may be credited to their simulated traders lacking foresight and being devoid of an incentive for intertemporal speculation.

Spulber (1996) proposes to introduce intermediacy as part of the basic framework of mainstream economics. He postulates a conceptual model where price-setting firms coordinate all trade and efficiently allocate resources. These market intermediaries are motivated by profit to set market clearing prices, effectively matching sales and purchases. These dealers' ability to extract gains from trade is measured by how accurately they anticipate shifts in supply and demand based on costly gathered and processed information. They stand ready to provide immediacy and liquidity in accordance with their expectations of arbitrage profits under the risk of uncertainty in market outcomes.

Rust and Hall (2003) presented a computational model of market intermediation that extends upon the framework proposed by Spulber (1996). This simulated market with endogenously determined microstructure introduces an additional type of intermediary to mediate trade, i.e., the *market-maker*. In this model buyers and sellers have the option of trading in an exchange operated by market-makers where prices are posted publicly. Or they can search for better offers to be negotiated privately among 'brokers' in a *dealer-market*. According to the simulated data, Rust and Hall conclude that the addition of a market-maker publicly posting prices strictly increases the expected gains from trade among buyers and sellers at the expense of the dealers' profit margins.

Both Rust and Hall (2003) and Spulber (1996) require all exchanges between buyers and sellers to be intermediated. Thus they fail to account for direct exchange, which is a substantial portion of market trade. Rust and Hall's model also fails to account for information asymmetry among all types of intermediaries, which implies homogeneity in forecasting abilities among intermediaries.

Miller et. al. (1977) conducted an empirical study to test whether seasonal markets can reach intertemporal competitive equilibrium under the same information conditions that yield competitive equilibrium in stationary markets. Based on the experimental evidence, they conclude that perfect foreknowledge of supply and demand is not a necessary condition for speculative markets to reach outcomes consistent with intertemporal competitive equilibrium predictions. Extending on his previous work, Spulber (1998) develops a model with a market microstructure where market-makers formulate bids and offers based on expectations of future market outcomes. They participate in a multilateral exchange market with publicly posted prices where buyers and sellers are able to trade directly with each other. Spulber postulates that competition among market-makers to gain an arbitrage advantage drives transactions costs down while increasing consumer and producer welfare.

The application of computational and experimental methods in economics has overcome the problems of general validity in empirical models. Simulated and human experiments allow a model to be simultaneously tested to large set of environmental, institutional and behavioral permutations. The robust results in these studies are transferable to a broader set of economies than those narrowly defined in microeconomic theories[8]. To wit, computational and experimental models have provided a wider array of market institutions and behavior that standard neoclassical theory discards as irrelevant or incompatible with competitive equilibrium. Upon a review of the available literature, however, these efforts fall short of providing a coherent explanation of dynamic process of competitive price formation in a decentralized mechanism of multilateral exchange. I introduce speculation into Smith's (1982) framework to determine decentralized intertemporal competitive price formation.

A Double Auction Market ACE Model

The model presented here maintains the essential governing rules of the Double Auction (DA) as proposed by Smith (1982). The DA has properties specifically suitable for testing propositions based on competitive price theory[9]. It provides a close characterization of multilateral exchange markets (i.e., securities and commodities), making it ideal to study the introduction of intermediation in speculative market.

In terms of a microeconomic system, a double auction market is composed of a microeconomic environment with private individual valuations and a microeconomic institution with a decentralized trading process. Traders posts bids and offers based on privately held beliefs about the valuations and publicly posted prices. In this computational rendition of the DA framework, the behavioral aspect attempts to model traders' abstract considerations based on institutional properties and communication rights subject to heterogeneous cognitive and

[8]Smith (1982) refers to this precept as *parallelism*. He states that "propositions about the behavior of individuals and the performance of institutions that have been tested in laboratory microeconomies apply also to nonlaboratory microeconomies where similar *ceteris paribus* conditions hold".

[9]Smith et. al. (1982) conducted a comparative study of competitive market institutions. They showed that the *double auction* mechanisms was far superior to all but a *unanimous voting tâtonnement* mechanism. The DA market outcomes are much nearer to competitive equilibrium predictions than all non-voting tâtonnement process and allocate resources far more efficiently. The efficiency drawback all for tâtonnement processes, as compared to DA, is that they are slower to converge to a competitive market price and "require more conditions to be fulfilled for an inefficient trade to occur"(Sunder, 2002).

budgetary constraints.

Based on ample experimental data, the performance of the baseline behavioral model – i.e., no intermediation – is measured by the proximity of the computational and experimental market outcomes under identical institutional arrangements. The extended intermediary model, where market-makers have arbitraging privileges, intends to capture intertemporal aspect in real-world market outcomes that remain unaccounted. To this end, the performance of this extended model is measured by the proximity of the computational market with microsimulated data of real-market outcomes.

Market

The market represents the microeconomic environment as proposed by Smith (1982). The microeconomic environment is the set of all initial endogenous attributes: list of goods, number and type of agents, agents' characteristics and initial endowments. In both computational and human experimental models of the DA market, the environment is a set of randomly generated market attributes. The environment is generated via a discrete event stochastic computer simulation – Monte Carlo simulation.

The market provides the necessary conditions for the DA to operate – i.e., number of trading days, length of market period, and so forth. Thus, it provides the set of initial circumstances that cannot be altered by the DA institution.

Controller

The controller represents the DA market institution. It is the *rule mechanism* responsible for assigning property rights over messages and the exchange of goods. At any point during the simulation, it acts as an automated auctioneer. It makes *'market-calls'* for a trading period to begin or end. It also calls for offers on a unit up for auction and announces the best offers on record.

At any point in a trading period, it takes bids or ask while there are units and capital available and trading offers converge to an acceptable price range. Upon opening a unit for auction, it announces a bidding price by a buyer and an asking price by a seller. Subsequently, any entering bid must be higher and any entering ask must be lower to be admitted. Once an offer – bid or ask – is reported, it becomes the standing offer and cannot be withdrawn. Specifically, an entering bid becomes the standing bid if it is higher than the last bid announced. Similarly, an entering ask becomes the standing ask if it is lower than the last ask announced. A sale contract is formed when the standing bid and standing ask overlap. The contract price is equal to the latest standing offer to be announced.

Once a contract occurs the auction for that unit ends. A new auction for another unit begins with the clearing of all previously posted auction bids and asks. All initial prices are reseted and this process continues until the end of a trading period. At the beginning of a new period all bids and asks are cleared. And the running average price is set to a midpoint between the highest possible ask and the lowest possible bid.

Agents

Based on a DA market structure, this model generates three types of agents: buyers, sellers, and market-makers.

Buyers and sellers are relatively unsophisticated trading agents. They exhibit *reinforcement learning* behavior in their offer-formulation strategies. Both are given initial endowments. The seller is endowed with goods and the buyer with capital. Each of them are also given an initial closed and bounded set of heterogenous valuations.

Buyers and sellers are unaware of others' valuations. And they do not account for others' actions in their strategic planning. Both of them, however, are able to recall previous trading prices and quantities within a period. They use this information to compute and track a running average of the market price.

Both buyers and sellers update their valuations by drawing from a independent and identically distributed set of random values. As trading progresses, this set of values converges to their true valuations. Both of them use a weighted sum of their converging valuations and running average price to formulate their offers. But only the buyers are constrained by their available account balances.

The weights given to each part of the weighted sum are representative of a tradeoff in offering strategies. Giving more weight to a converging set of values exhibits *myopic behavior*[10], ignoring market information. Giving more weight to the running average price exhibits *herding behavior*[11], relying heavily on market information.

Market-makers are more sophisticated trading agents. They exhibit a hybrid of *reinforcement learning* and *belief-based learning* behavior. They consider market pricing as well as their own estimated expectations of forecasted market-clearing prices. Thus, market-makers are able to arbitrage of their (adjusted) expectations whenever these are more profitable than the running average market price.

[10]Myopic behavior reflects nearsightedness in strategic decision making where agents assign more weight to their own individual beliefs about valuation.

[11]Herding behavior reflects mimicry in strategic decision making where agents assign more weight to moving average market price.

Buyers

The buyer has one set of values for all the units he wants to purchase within a given trading period. This set is represented by a one dimensional array. The size of this array is equal to the number of desired units for that period.

At the beginning of every period, the buyer chooses his initial bid, b_0, from an independent and identically distributed set of random values. This set of values is skewed towards a neighborhood of the minimum price the buyer designates for the current period, P_{min}. Lacking information about the market trading price, this strategy reflects an initial tendency to drive a hard bargain based solely on individual valuations:

$$b_0 = \left(1 - e^{-\phi(1-\phi)}\right) \cdot (v - P_{min}) + P_{min} \tag{1}$$

where v is the induced value and $\phi \sim U\left[0,1\right]$ is a uniformly distributed random number such that $b_0 \sim U\left[P_{min}, \frac{3P_{min}+v}{4}\right]$. Once trading starts, the buyer's bid formulation changes to include publicly posted market information in his dynamically adjusted strategy. At any point in his individual sequence of posted bids, s, the buyer formulates a bid, b_s, as a tradeoff between a dynamically adjusted set of individual valuations and the market record of trading prices in the current period:

$$b_s = \min \left\{ \alpha\, U_s + (1-\alpha) \cdot \frac{\left(\hat{P} + v^s\right)}{2}, x_s \right\} \tag{2}$$

where $\alpha \in (0,1)$ is an anchoring weight[12], $\hat{P}$ is the moving average of the trade contract prices in the current market period and x_s is the remaining purchasing balance. The buyer's converging willingness-to-pay (WTP), U_s, is an independent and identically distributed random number within a set of converging valuations. This set of valuations is slightly skewed towards the converging unit value, v^s, to reflect an increasing willingness to settle:

$$U_s = e^{-8\phi^2(1-\phi)^2} \cdot (v^s - P^s_{min}) + P^s_{min} \tag{3}$$

where $\phi \sim U\left[0,1\right]$ is a uniformly distributed random number such that $U_s \sim \left[\frac{3v^s+2P^s_{min}}{5}, v^s\right]$. The converging minimum price, P^s_{min}, represents a dynamically adjusted lower bound of the set of "acceptable" bid prices. It approaches the buyer's trade-unit value, v, at a subjective rate of convergence determined by the buyer's number of tradable units remaining and the buyer's number of bids posted:

$$P^s_{min} = v - (1 - \lambda_{s,n}) \cdot (v - P_{min}) \tag{4}$$

[12]The *anchoring weight* refers to the tendency of a buyer or seller to rely on their converging valuations.

where $\lambda_{s,n} \to 0^+ \Rightarrow P^s_{min} \to P_{min}$ and $\lambda_{s,n} \to 1 \Rightarrow P^s_{min} \to v$. The converging trade-unit value, v^s, represents a dynamically adjusted upper bound of the set of "acceptable" bid prices – or adjusted reservation bidding price. It approaches the buyer's trade-unit value, v, at a subjective rate of convergence determined by the buyer's number of tradable units remaining and the buyer's number of bids posted:

$$v^s = v - \frac{1}{2}(1 - \lambda_{s,n}) \cdot (v - P_{min}) \tag{5}$$

where $\lambda_{s,n} \to 0^+ \Rightarrow v^s \to \frac{1}{2}(v + P_{min})$ and $\lambda_{s,n} \to 1 \Rightarrow v^s \to v$. As both converging variables approach the trade-unit value, U_s provides a contraction mapping that collapses into a small neighborhood around the trade-unit value. Thus, as trading progresses, the buyer's bid approaches a weighted sum of the buyer's trade-unit value and the moving average of the current period's trading prices.

Seller

The seller has one set of costs for all units she has for sale within a given trading period. This set is represented by a one dimensional array. The size of this array is equal to the number of units for sale for that period.

At the beginning of every period, the seller chooses her initial ask, a_0, from an independent and identically distributed set of random values. This set of values is skewed towards a neighborhood of the maximum price the seller designates for the current period, P_{max}. Lacking information about the market price, this strategy reflects an initial tendency to drive a hard bargain based solely on individual valuations:

$$a_0 = c + e^{-\phi(1-\phi)} \cdot (P_{max} - c) \tag{6}$$

where c is the induced cost and $\phi \sim U[0,1]$ is a uniformly distributed random number such that $a_0 \sim \left[\frac{c+3P_{max}}{4}, P_{max}\right]$. Once trading starts, the seller's ask formulation changes to include publicly posted market information in her dynamically adjusted strategy. At any point in her individual sequence of posted asks, s, the seller formulates an ask, a_s, as a tradeoff between a dynamically adjusted set of individual valuations and the market record of publicly posted trading prices for the current period:

$$a_s = \alpha W_s + (1 - \alpha) \cdot \frac{\left(\hat{P} + c^s\right)}{2} \tag{7}$$

where $\alpha \in (0,1)$ is an anchoring weight and $\hat{P}$ is the moving average of the trade contract price in the current trading period. The seller's converging willingness-to-accept (WTA), W_s, is an independent and identically distributed random number with a set of converging valuations. This set of valuations is slightly skewed towards the converging unit cost, c^s, to reflect an increasing willingness to settle:

$$W_s = P^s_{max} - e^{-8\phi^2(1-\phi)^2} \cdot (P^s_{max} - c^s) \tag{8}$$

where $\phi \sim U[0,1]$ is a uniformly distributed random number such that $W_s \sim \left[c^s, \frac{3c^s+2P^s_{max}}{5}\right]$. The converging maximum price, P^s_{max}, represents a dynamically adjusted upper bound of the set of "acceptable" ask prices. It approaches the seller's trade-unit cost, c, at a subjective rate of convergence determined by the seller's number of tradable units remaining and the seller's number of asks posted:

$$P^s_{max} = c + (1 - \lambda_{s,n}) \cdot (P_{max} - c) \tag{9}$$

where $\lambda_{s,n} \to 0^+ \Rightarrow P^s_{max} \to P_{max}$ and $\lambda_{s,n} \to 1 \Rightarrow P^s_{max} \to c$. The converging trade-unit cost, c^s, represents a dynamically adjusted lower bound of the set of "acceptable" ask prices – or adjusted reservation asking price. It approaches the seller's trade-unit cost, c, at a subjective rate of convergence determined by seller's number of tradable units remaining and her individual number of posted asks:

$$c^s = c + \frac{1}{2}(1 - \lambda_{s,n}) \cdot (P_{max} - c) \tag{10}$$

where $\lambda_{s,n} \to 0^+ \Rightarrow c^s \to \frac{1}{2}(c + P_{max})$ and $\lambda_{s,n} \to 1 \Rightarrow c^s \to c$. As both converging variables approach the trade-unit cost, W_s, provides a contraction mapping that collapses into a small neighborhood around the trade-unit cost. Thus, as trading progresses, the seller's ask approaches a weighted sum of the seller's trade-unit cost and the moving average of the current period's trading prices.

Subjective rate of convergence, $\lambda_{s,n}$

The individual process by which each buyer and seller agent converges to his own true valuations reflects an increasing *willingness-to-settle* subject to unsuccessful attempts to trade. It represents a truth-revealing mechanism with a convergence speed proportional to the number of offers entered and the number of remaining units intended for trade.

The subjective rate of convergence, $\lambda_{s,n}$, is a Gaussian function of an agent's individual individual sequence of trade offerings, s, and the remaining number of units desired to trade, n:

$$\lambda_{s,n} = \exp\left\{\frac{-\left(n + \bar{n}^2\right)}{\left(s^2 + \bar{n}^2\right)^{1/2} + 2n}\right\} \tag{11}$$

where $\bar{n}$ is the number of traded units, such that $\bar{n} \propto \frac{1}{n}$. Generally,

$$\lim_{s \to \infty} \lambda_{s,n} = 1 \tag{12}$$

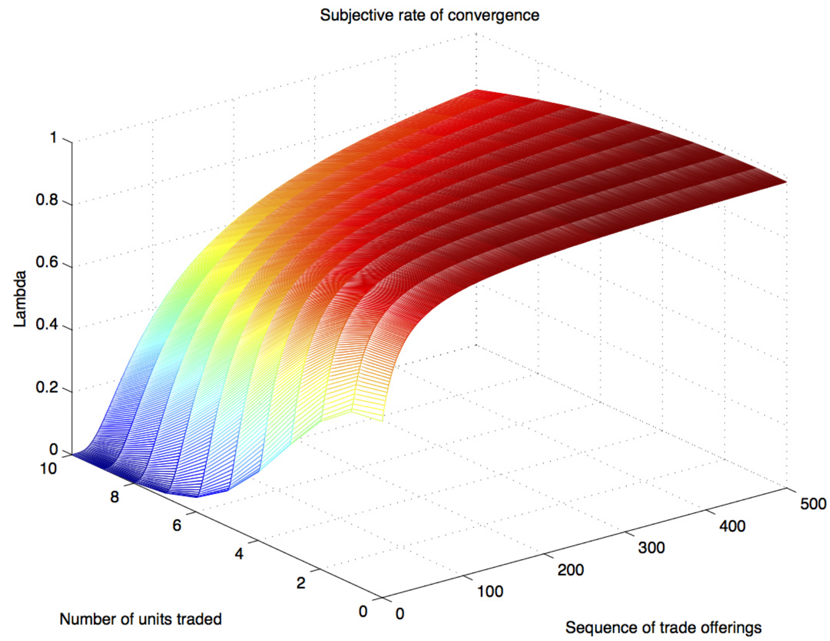


Figure 1: Subjective rate of convergence, $\lambda_{s,n}$, $\bar{n} \in [0, 10]$, $s \in [0, 500]$

but as shown in the figure below, $\lambda_{s,n} \to 1$ much faster as $\bar{n} \to 0^+$.

In this case with 10 tradable units and within a window of 500 posted offerings

$$\lim_{\substack{s\to 500\\ \bar{n}\to 0^+}} \lambda_{s,n} > \lim_{\substack{s\to 500\\ \bar{n}\to 10}} \lambda_{s,n} \tag{13}$$

where $\lambda_{500,0} \simeq 1$ and $\lambda_{500,10} \simeq 0.8$. As his individual sequence of offers to trade increases an agent will tend reveal his true valuations, regardless of the number of units traded. But with a larger number of tradable units remaining in his possession, not yet traded, the agent will tend to reveal his true valuations at a faster rate. This intends to reflect a *sense of desperation* proportional to the potentially foregone gains from trade.

Market-maker

The market-maker is a more 'sophisticated' trading agent. Unlike the buyers and the sellers, he is able to aggregate information across periods to capture an otherwise forgone intertemporal profit[13]. That is, the market-maker has a

[13] In accordance with observations by Plott and Sunder (1982), market-makers competing to earn an arbitrage profit are able to aggregate and disseminate their privileged information

'memory' of past outcomes, a knowledge of theoretically predicted outcomes, and an ability to forecast future outcomes.

Also unlike the buyers and sellers, the market-maker is not endowed with goods or capital at the beginning of each trading period.

Instead, each market-maker is endowed with the set of theoretically predicted market-clearing prices and quantities for all trading periods. Each market-maker also has an ability to formulate a limited set of forecasted market-clearing prices – with some degree of uncertainty. And he is also able to observe the standing bids and asks in the current period.

At any point in a trading period, the market-maker can formulate a bid or an ask based on his knowledge of the standing offers. If his discounted value for the current period t, $\tilde{V}^t$, is greater than the standing ask plus a small amount

$$\tilde{V}^t > a^* + \delta \tag{14}$$

then he places a bid equal to the standing ask, such that

$$b_s = a^* \tag{15}$$

Else, he chooses a bid from a uniformly distributed closed set of random values. This set of values is bounded below by the standing ask. And it is bounded above by the standing ask plus a small number:

$$b_s \sim U\left[a^*, a^* + \delta\right] \tag{16}$$

Conversely, if his discounted cost for the current period $t \in \{1, 2, ..., T\}$, $\tilde{C}^t$, is less than the standing bid minus a small amount

$$\tilde{C}^t < b^* - \delta \tag{17}$$

then he places an ask equal to the standing bid, such that

$$a_s = b^* \tag{18}$$

Otherwise, he chooses an ask from a uniformly distributed closed set of random values. This set is bounded below by the standing bid minus a small amount. It is bounded above by the standing bid:

$$a_s \sim U\left[b^* - \delta, b^*\right] \tag{19}$$

Using the sets of theoretically predicted and forecasted outcomes, the market-maker formulates his discounted value for period t:

$$\tilde{V}^t = \max\left\{(1+r)^{-\tau} E\left[\underline{p_t}\right] + i\,\mathbf{p}_\tau, \mu^p_{\frac{1}{2}}\right\} \tag{20}$$

about the current and future states of nature due to the institutional features of the DA market. See Plott and Sunder (1982) and Plott (1986) for further discussion.

and his discounted cost for period t:

$$\tilde{C}^t = \min\left\{(1+r)^\tau\, E\left[\underline{p_t}\right] - i\,\mathbf{p}_\tau, \mu^p_{\frac{1}{2}}\right\} \tag{21}$$

where r is the interest rate, τ is the number of periods of foresight into the future, i is the inventory cost, and $\mu^p_{\frac{1}{2}}$ is the median of the set of theoretically predicted market-clearing prices.

For all periods t such that $t+\tau \leq T$, the market-maker uses a subset of theoretically predicted market-clearing prices, $\underline{p_t} = \{p_{t+1}, p_{t+2}, \ldots, p_{t+\tau}\}$, to estimate an expected clearing price over the range of periods of foresight:

$$E\left[\underline{p_t}\right] = \frac{1}{\tau}\sum_{k=t+1}^{t+\tau} p_k \tag{22}$$

For any period t given a τ periods of foresight, such that $t+\tau > T$, the estimated market-clearing price is

$$E\left[\underline{p_t}\right] = \frac{1}{\tau}\sum_{k=T-\tau}^{T} p_k \tag{23}$$

At either discount valuation, this estimate is multiple by a compounded interest rate. And the set of forecasted market-clearing prices, $\{\mathbf{p}_t, \mathbf{p}_{t+1}, \ldots, \mathbf{p}_\tau\}$, is multiplied by the inventory cost. The sequence of forecasted market clearing prices is constructed using a *double exponential smoothing process*. This allows to correct for trend as well as forecasted past the current period t:

$$\begin{aligned} \mathbf{p}_1 &= p_1 \\ z_1 &= p_2 - p_1 \end{aligned} \tag{24}$$

for $t > 1$

$$\begin{aligned} \mathbf{p}_t &= \gamma \hat{P}_t + (1-\gamma)\left(\mathbf{p}_{t-1} + z_{t-1}\right) \\ z_t &= \varphi\left(\mathbf{p}_t - \mathbf{p}_{t-1}\right) + (1-\varphi)\, z_{t-1} \end{aligned} \tag{25}$$

where $\gamma \in (0,1)$ is the *data smoothing factor*, $\varphi \in (0,1)$ is the trend smoothing factor and $\hat{P}_t$ is the final moving average trading price of the previous period, $t-1$. To forecast past the last recorded moving average trading price for $t+\tau$, we construct a series of estimates:

$$\mathbf{p}_{t+\tau} = \mathbf{p}_t + \tau z_t \tag{26}$$

The discounted value is set equal to the maximum value between a sum of the weighted estimated and forecasted prices, and the set of theoretically predicted prices. Similarly, the discounted cost is set equal to the minimum value between a subtraction of the weighted estimated and forecasted prices, and the set of theoretically predicted prices.

The market-maker's profits from his offers are the margins between the standing offers and his discounted valuations.

Results

Baseline model with no market-maker

Figure 2 shows the series of contract prices of the trading periods for multiple simulated realities of a DA market without market-makers. The box-plot measurements provide a visually discernible distribution of the trade contract prices among buyers and sellers in the agent-based simulated market. Each point in the scatter plot represents the theoretically predicted equilibrium price within a trading period. The line plot represents theoretically predicted intertemporal equilibrium prices adapted from the formulation used by Miller et al. (1977)[14].

[14] Miller et. al. (1977) use an extension of spatial price equilibrium theory for a DA market with unknown seasonal variants. For seasons $i = \{1, 2\}$ with supply $S_i(p_i)$ and demand $D_i(p_i)$, let p_i^* be its (*intratemporal*) equilibrium price such that $D_i(p_i^*) = S_i(p_i^*)$. Suppose T is the carry over cost from season one to season 2 such that $p_2^* > p_1^* + T$. They define a *pair of intertemporal equilibrium prices*, (p_1^0, p_2^0), such that the following market conditions are satisfied:

$$p_2^0 = p_1^0 + T, \qquad S_1(p_1^0) - D_1(p_1^0) = D_2(p_2^0) - S_2(p_2^0)$$

I adapted their formulation with no carryover costs, $T = 0$, to define theoretical intertemporal equilibrium prices between any two periods, $i, i+1$:

$$p_{i+1}^0 = p_i^0, \qquad S_i(p_i^0) - D_i(p_i^0) = D_{i+1}(p_{i+1}^0) - S_{i+1}(p_{i+1}^0)$$

where speculative carryover is motivated by an arbitrage opportunity from one period to the next.

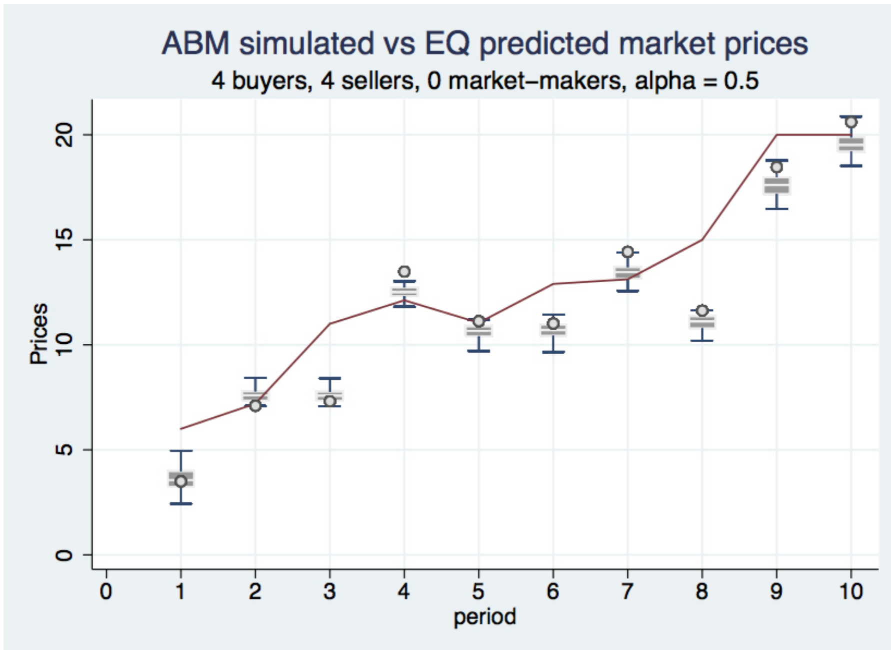


Figure 2: Agent-based simulated vs. theoretically predicted trade contract prices (no market-maker)

In spite differences in their microstructures, this baseline model exhibits similar behavior to Gode and Sunder (1993) ZI-C model. The set of market environment attributes for each period follow a stochastic process with a drift (i.e., random walk). Coupled with a lack of memory of events from previous periods, the series of contract prices manifests a lack of intertemporal learning. Additionally, imposing intratemporal memory (i.e., moving average trading price) and converging valuations in the formulation of offerings manifests intratemporal learning.

This baseline model exhibits a volatility of prices and convergence to equilibrium much closer to price series in human experiments than Gode and Sunder (1993). Each period's set of market contract prices closely converges around its corresponding competitive equilibrium price – from above or below, according to the shape of the supply and demand curves. Table 1 outlines the interquartile range, median and mean contract prices along with the *coefficient of variation* (cv) [15], as well as the theoretically predicted intratemporal and intertemporal equilibrium prices.

[15]The coefficient of variation, or cv, is a standardize measure of dispersion defined as the ratio of the standard deviation to the mean, $\frac{\sigma}{\mu}$. It shows the extent of variability with respect to the mean of the population.

	ABM trade contract prices						EQ predicted prices	
period	N	mean	cv	p25	p50	p75	intra-temporal	inter-temporal
1	72	3.591	.1534	3.241	3.5550	3.983	3.5	6
2	96	7.652	.0525	7.347	7.591	7.780	7.1	7.2
3	120	7.604	.0437	7.342	7.563	7.767	7.31	11
4	58	12.491	.0269	12.318	12.491	12.728	13.488	12.12
5	59	10.597	.0364	10.407	10.664	10.877	11.126	11.05
6	70	10.662	.0388	10.436	10.691	10.956	11.018	12.9
7	62	13.449	.0317	13.146	13.458	13.693	14.428	13.12
8	65	11.074	.0332	10.792	11.104	11.357	11.624	15
9	50	17.563	.0298	17.185	17.598	17.973	18.46	20
10	44	19.550	.0284	19.214	19.496	19.877	20.61	20

Table 1: No market-maker simulated trade contract vs. theoretically predicted prices

The simulated DA market data set shows each period's contract prices tightly distributed about the mean. The set of contract prices from the first period shows the largest measure of dispersion with a coefficient of variation of .15345 – such that $(\mu_1, \sigma_1) = (3.591, .5510)$. The rest of the periods have measures of dispersion three to five times smaller. Additionally, the mass of the distribution for each period's contract prices seems skewed towards its corresponding intratemporal equilibrium price while only incidentally coinciding with the intertemporal equilibrium prices.

Model with a single market-maker

Figure 3 shows the series of contract prices of the trading periods for multiple simulated realities of a DA market with a single market-maker able to make forecasts with four periods of foresight. In contrast with the baseline model, the addition of a market-maker formulating offerings as a function of observed market prices in previous periods and estimates of future equilibrium prices manifest intertemporal learning in the series of trade contracts.

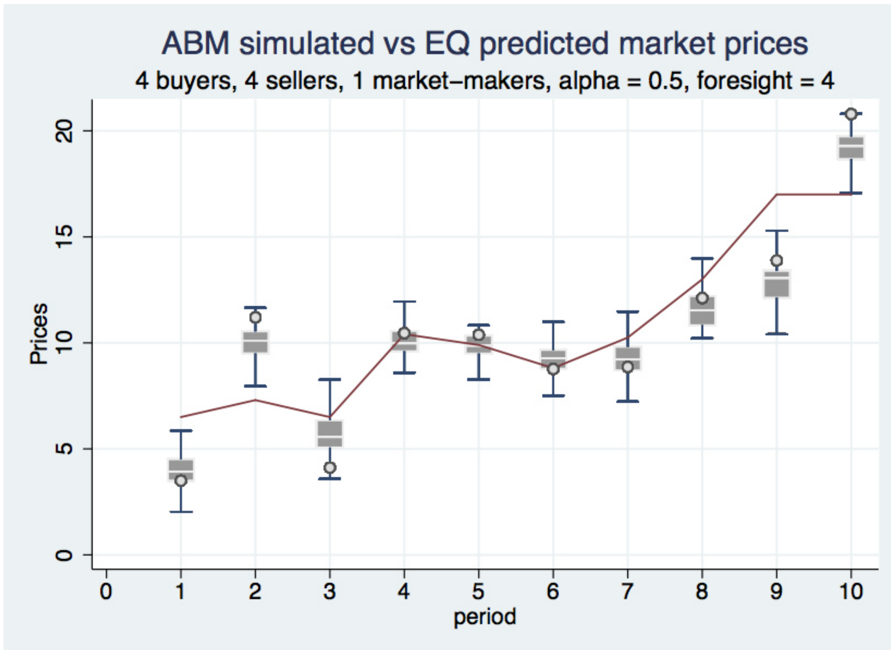


Figure 3: Agent-based simulated vs. theoretically predicted trade contract prices (single market-maker)

This modified DA market with a single market-maker exhibits greater volatility of prices than the baseline model. Each period's set of market prices does not converge as tightly around the corresponding intratemporal equilibrium price. Conversely, the overall distribution of market contract prices follows the intertemporal equilibrium prices closer than the baseline. The series of interquartile ranges show the central tendency of market contract prices converging to the corresponding intratemporal equilibrium prices.

Table 2 shows a greater measure of dispersion of trade contract prices with the addition of a single market-maker. Outside the first period with a coefficient of variation of .1982, the measure of dispersion in each period of this single market-maker model is two to four times larger than the measure in the corresponding period in the baseline model.

	ABM trade contract prices						EQ predicted prices	
period	N	mean	cv	p25	p50	p75	intra-temporal	inter-temporal
1	104	3.948	.1982	3.506	3.921	4.510	3.5	6.5
2	107	9.949	.0906	9.506	10.102	10.543	11.199	7.3
3	111	5.710	.1836	5.067	5.553	6.346	4.114	6.5
4	130	10.127	.0678	9.607	9.968	10.561	10.460	10.41
5	120	9.861	.0582	9.498	9.851	10.337	10.378	9.9
6	117	9.234	.0765	8.782	9.267	9.665	8.77	8.81
7	125	9.278	.1021	8.706	9.212	9.813	8.86	10.25
8	152	11.591	.0797	10.832	11.529	12.207	12.116	13
9	158	12.759	.0747	12.133	13.039	13.394	13.891	17
10	111	19.087	.0569	18.674	19.268	19.736	20.793	17

Table 2: Single market-maker simulated trade contract vs. theoretically predicted prices

Similarly to baseline model, this data set shows the bulk of the series for each period's contract prices skewed towards its corresponding intratemporal equilibrium price. Unlike the baseline model, there is a higher incidence of the series of intertemporal prices falling into the higher or lower quartiles of the series of trade contract prices.

Figure 4 shows the series of trade contract prices separated by market-maker participation for each market period over 12 simulated realities. The blue scatter plots depict the series of trade prices where neither contracting party was a market-maker. The red scatter plots depict the series of trade prices where one of the contracting parties was a market-maker. For each period's scatter plot, the blue and red vertical lines mark the intratemporal and intertemporal equilibrium prices, respectively.

In spite of some overlap among the two series of trade contract prices, the blue series (no market-maker) converges to values nearer to the intratemporal equilibrium while the red series (market-maker) converge to values nearer the intertemporal equilibrium. Outside the last two periods where boundary issues in forecasting[16] manifest in the red series of trade contract prices, the difference in trends is highlighted where intratemporal and intertemporal equilibrium prices are markedly different (i.e., periods one, two, three and seven).

[16] A market-maker estimates the market-clearing price at time t as an average of predicted competitive equilibrium prices for the next τ periods. For any period t such that the next τ periods exceed the total number periods, $t+\tau > T$, the market-clearing price estimation averages the predicted competitive equilibrium prices from period $T-\tau$ to period T. Specifically, the single market-maker estimates the market-clearing price in periods six to ten using the same average of the predicted competitive equilibrium prices for periods six to ten. Similarly, the hypothetically predicted intertemporal equilibrium price uses the same midpoint value for second to last and last periods – i.e., periods nine and ten.

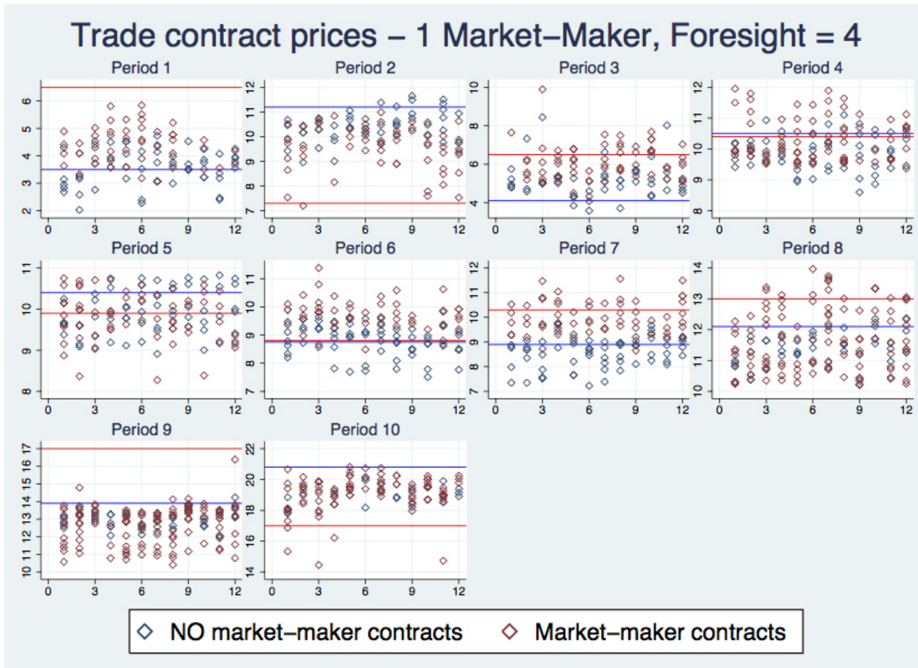


Figure 4: ABM trade contract prices by market-maker participation in trade transaction

Table 3 outlines the interquartile range, dispersion and central tendency of the series of trade contract prices for each period parsed out by whether one of the transacting parties was a market-maker or not. Notwithstanding boundary issues in the last two periods, the measures of central tendency and the mass of the distribution of the two series converge more noticeably apart for periods where the intratemporal equilibrium and intertemporal (predicted) equilibrium prices are further apart.

For periods one, two, three and seven where the difference between predicted equilibrium prices is greater there is less overlap between the two series of trade contract prices.The interquartile range overlap between the two series in periods one and two is limited to a single quartile. There is no such overlap between the two series in periods three and seven. In all four of these periods, the mean and median of the series of trade prices where neither contracting party was a market-maker converge to values closer to the intratemporal equilibrium price than the series of all trade contracts. The mean and median of the series of trade prices where one of the contracting parties was a market-maker converge to values closer to the intertemporal equilibrium price than the series of all trade contracts. In contrast, for the rest of the periods the interquartile range overlap between the two series is nearly complete and the mean and median prices converge to values closer together.

		ABM trade contract prices						EQ predicted prices	
period		N	mean	cv	p25	p50	p75	intra-temporal	inter-temporal
1	No MM	57	3.578	.1836	3.167	3.656	4.078	3.5	6.5
	MM	47	4.398	.1557	3.897	4.419	4.875		
2	No MM	52	10.375	.0589	10.016	10.413	10.746	11.199	7.3
	MM	55	9.546	.0996	8.939	9.733	10.307		
3	No MM	49	5.154	.1806	4.667	5.078	5.311	4.114	6.5
	MM	62	6.150	.1504	5.444	6.066	6.732		
4	No MM	51	9.860	.0590	9.433	9.911	10.257	10.460	10.41
	MM	79	10.300	.0676	9.710	10.032	10.739		
5	No MM	57	9.898	.0559	9.491	9.832	10.371	10.378	9.9
	MM	63	9.828	.0604	9.505	9.868	10.278		
6	No MM	67	8.848	.0632	8.512	8.956	9.235	8.77	8.81
	MM	50	9.750	.0548	9.418	9.790	9.960		
7	No MM	66	8.610	.0714	8.209	8.753	9.033	8.86	10.25
	MM	59	10.025	.0645	9.569	9.865	10.554		
8	No MM	43	11.357	.0471	10.970	11.379	11.687	12.116	13
	MM	109	11.683	.0878	10.810	11.599	12.513		
9	No MM	40	12.967	.0392	12.608	13.049	13.266	13.891	17
	MM	118	12.688	.0831	11.820	13.028	13.412		
10	No MM	21	19.148	.0323	18.849	19.206	19.571	20.793	17
	MM	90	19.072	.0614	18.609	19.317	19.785		

Table 3: Single market-maker distribution of trade contract prices by market-maker participation

Table 3 shows that coefficients of variation for the series of trade prices where neither contracting party was a market-maker are strictly smaller than the corresponding coefficients of variation in the series of all trade contract prices in Table 2. In comparison, the coefficient of variation in the series of trade prices where one of the contracting parties is a market-maker is smaller than the coefficient of variation in the series of all trade contract prices in periods one, three, six and seven, nearly the same in period four, and greater everywhere else.

Model with two market-makers

Figure 5 shows the series of contract prices of the trading periods for multiple simulated realities of a DA market with two market-makers able to make forecasts with different foresights. In keeping with the previous version, the addition of market-makers formulating offers as weighted sums of previous periods' market prices and estimated future equilibrium prices manifests intertemporal learning in the series of trade contracts.

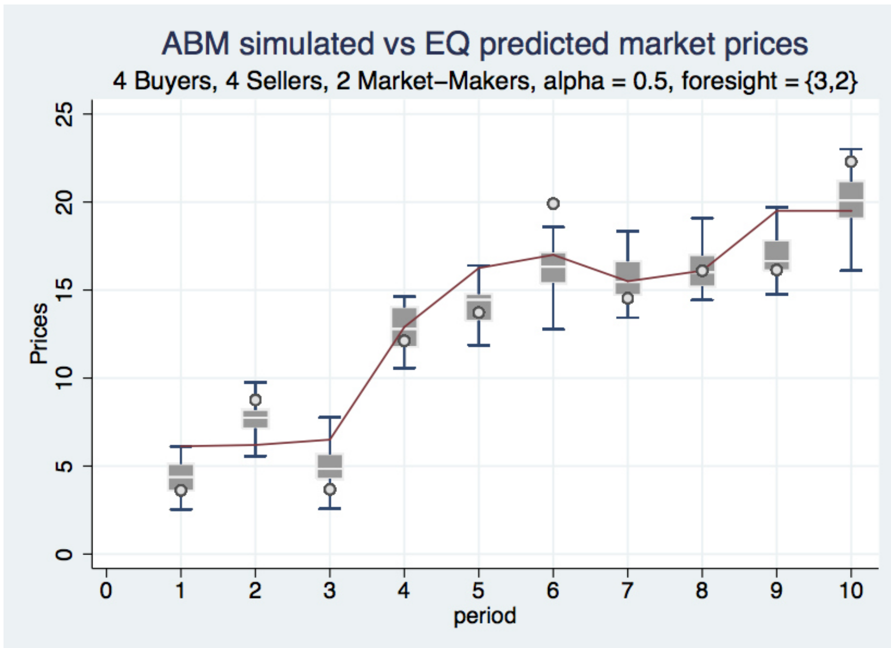


Figure 5: Agent-based simulated vs. theoretically predicted trade contract prices (dual market-maker)

This dual market-maker version exhibits greater volatility than the baseline and single market-maker versions of the DA market model. Similar to the single market-maker version, the distribution of market prices for each period does not converge tightly around the corresponding intratemporal prices. In general the interquartile ranges exhibit measures of central tendency of trade prices converging to values close to the intratemporal equilibrium prices while the overall series of trade prices follows the intertemporal equilibrium prices much closer than both of the previous versions.

Table 4 shows a noticeably greater measure of dispersion in the series of trade contract prices for the dual market-maker version of the DA market than the baseline. Outside the series of trade contract prices for periods seven, eight and nine, the measures of dispersion for this version are larger than the corresponding measures of dispersion in the single market-maker version. In period seven the double market-maker version shows a coefficient of variation of .0774, which is clearly smaller than the corresponding coefficient of variation of .1021 for the single market-maker. For periods eight and nine, however, the coefficients of variation are very similar with differences in a 10^{-3} range.

	ABM trade contract prices						EQ predicted prices	
period	N	mean	cv	p25	p50	p75	intra-temporal	inter-temporal
1	123	4.378	.2150	3.603	4.357	5.117	3.625	6.125
2	251	7.683	.1376	7.140	7.738	8.187	8.749	6.2
3	147	4.988	.2102	4.281	4.846	5.677	3.678	6.5
4	130	12.815	.0923	11.770	12.766	14.022	12.12	12.9
5	149	14.099	.0656	13.259	14.427	14.775	13.723	16.25
6	127	16.135	.0843	15.394	16.313	17.141	19.907	17
7	169	15.723	.0774	14.722	15.449	16.637	14.535	15.5
8	170	16.227	.0704	15.204	15.992	16.979	16.095	16.1
9	174	17.022	.0711	16.121	16.649	17.818	16.144	19.5
10	172	20.063	.0716	19.075	20.076	21.177	22.297	19.5

Table 4: Dual market-maker simulated trade contract vs. theoretically predicted prices

With the exception of period six, these results show the bulk of the series for each period's trade contract prices skewed towards the corresponding intratemporal equilibrium price. However, the interquartile ranges show a greater measure of dispersion from the median that coincide a lot closer with the intertemporal equilibrium price than the single market-maker version.

Figure 6 shows the series of trade contract prices separated by market-maker participation for each market period over 12 simulated realities. The blue scatter plots depict the series of trade prices where neither contracting party was a market-maker. The red scatter plots depict the series of trade prices where one of the contracting parties was a market-maker. For each period's scatter plot, the blue and red vertical lines mark the intratemporal and intertemporal equilibrium prices, respectively.

In contrast with Figure 4, there is an inordinate amount of trades where one of the contracting parties was a market-maker. Only the first four periods show any discernible pattern of trades where neither contracting party was a market-maker. For period two the series of trades where neither contracting party was a market-market is subsumed by the sheer volume of trades where one of the contracting parties was a market-maker. Nevertheless, a pattern can be discerned where both series cluster together to form darker-blue (near purple) contrast. For periods one and three the blue series (no market-maker) converges towards values nearer the intratemporal equilibrium while the red series (market-maker) converges towards values nearer the intertemporal equilibrium. Period four already shows a disproportionately larger number of contracts with a market-maker with an intermittent trend difference with respect to contracts made with no market-maker – i.e., runs one, four and seven.

For periods five to nine the ratio of trades where one of the contracting parties was a market-maker to trades where neither contracting party was a market-maker ranges from 7:1 to 14:1. For period ten only in three out of 172 trades neither contracting party was a market-maker. Similar to plots for the single market-maker version, the red series of trade prices converges closer towards values nearer the intertemporal equilibrium price where it is further apart from the intratemporal equilibrium price. When the intratemporal and intertemporal equilibrium prices are closer together, the red series shows greater dispersion and no clear converging trend.

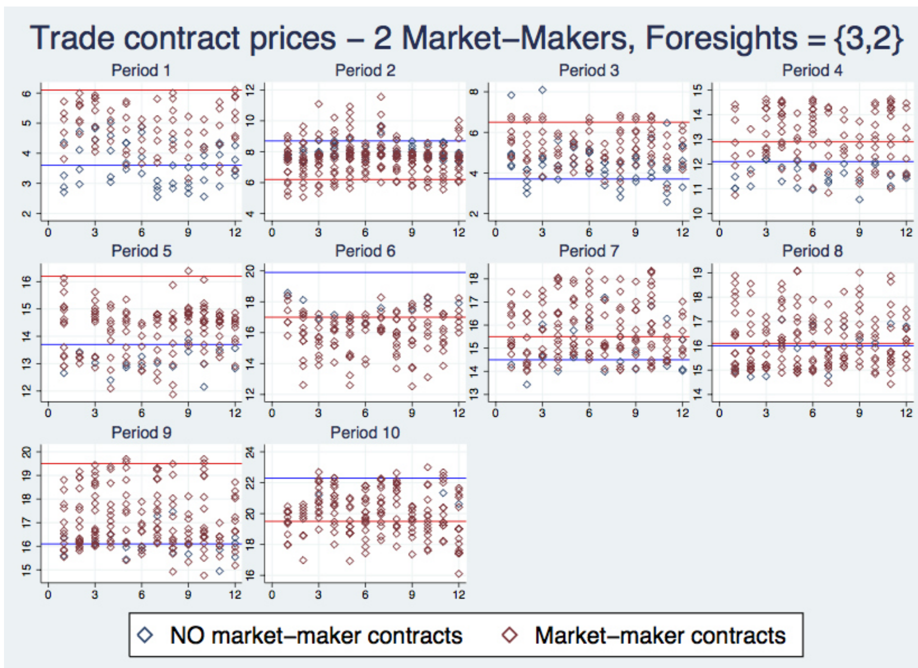


Figure 6: ABM trade contract prices by market-maker participation in transaction

Table 5 shows measures of central tendency and dispersion that exhibit a crowding effect as a result of the inordinate amount of trades where one of the parties was a market-maker. Most of the periods continue to show convergence of the two series of trade contracts consistent with the patterns observed in the single market-maker version of the model. However, periods two and four show a reversal in the tendency for the two series to converge noticeably apart for periods where the intratemporal and intertemporal equilibrium prices are further apart. To wit, in period two the overwhelming density of the widely disperse distribution of trades where one of the contracting parties was a market-maker nearly subsumes the distribution of trades where neither contracting party was a market-maker. Also the bulk of the series converge very close together. In period four, where the intratemporal and intertemporal equilibrium prices a closer together, the mass of the distribution of the two series converge further apart and there is no overlap between their interquartile ranges.

		ABM trade contract prices						EQ predicted prices	
period		N	mean	cv	p25	p50	p75	intra-temporal	inter-temporal
1	No MM	49	3.674	.1980	3.013	3.576	4.337	3.625	6.125
	MM	74	4.845	.1572	4.276	4.883	5.479		
2	No MM	50	8.052	.0621	7.708	7.926	8.384	8.749	6.2
	MM	201	7.591	.1498	6.875	7.678	8.011		
3	No MM	57	4.419	.2324	3.767	4.354	4.792	3.678	6.5
	MM	90	5.349	.1673	4.717	5.335	6.028		
4	No MM	27	11.526	.0381	11.105	11.543	11.938	12.12	12.9
	MM	103	13.153	.0820	12.335	13.171	14.209		
5	No MM	18	13.055	.0325	12.860	13.003	13.427	13.723	16.25
	MM	131	14.243	.0619	13.593	14.527	14.823		
6	No MM	12	17.653	.0363	17.048	17.730	18.184	19.907	17
	MM	115	15.977	.0825	15.203	16.174	16.951		
7	No MM	19	14.962	.0737	14.084	14.592	16.027	14.535	15.5
	MM	150	15.819	.0759	14.819	15.506	16.760		
8	No MM	12	15.991	.0592	14.863	16.182	16.828	16.095	16.1
	MM	158	16.245	.0711	15.209	15.982	17.021		
9	No MM	20	16.070	.0362	15.698	16.030	16.365	16.144	19.5
	MM	154	17.146	.0709	16.170	16.737	18.085		
10	No MM	3	21.053	.0180	20.621	21.204	21.334	22.297	19.5
	MM	169	20.045	.0720	19.067	20.066	21.082		

Table 5: Double market-maker distribution of trade contract prices by market-maker participation

With the exception of period three, Table 5 shows that the coefficients of variation for the series of trade prices where neither contracting party was a market-maker are smaller than the corresponding coefficients of variation for the series of all trade contract prices in Table 4. Except for period one with a coefficient of variation of .198 for this series, the rest of periods have coefficients of variation two to three times smaller than the corresponding coefficients of variation for the series of all trade contracts.

In comparison, aside from period two the coefficients of variation for the series of trade prices where one of the contracting parties was a market-maker are slightly smaller than the corresponding coefficients of variation for the series of all trade contract prices. For periods one, three and four, the difference between the coefficients of variation is in the range of 10^{-1} to 10^{-2}. For periods five to ten this difference narrows to range of 10^{-3} to 10^{-4} , which by comparison are nearly indistinguishable.

Comparing the models for goodness of fit and efficiency

Table 6 outlines the coefficients of determination[17] for the series of trade contract prices over 12 simulated realities of each DA market version. The top number on each cell represents the R^2 between the observed trade prices in the simulated market and the theoretically predicted equilibrium. The bottom number in parenthesis represents the *root mean squared error* (RMSE), which can be interpreted as the standard deviation between the observed simulated trade prices and equilibrium predictions.

	All trades		Trades w/o MM		Trades w/ MM	
Version	intra	inter	intra	inter	intra	inter
No MM	.9849 (.5167)	.8429 (1.664)	–	–	–	–
Single MM	.9325 (.9805)	.6995 (2.067)	.9594 (.6855)	.6691 (1.957)	.9179 (1.098)	.6926 (2.124)
Dual MM	.8911 (1.719)	.8903 (1.725)	.9662 (.9338)	.8591 (1.906)	.8644 (1.808)	.8896 (1.631)

Table 6: Coefficients of determination (R^2) for intratemporal and intertemporal equilibrium prices

Table 6 shows that the simulated outcomes in the baseline model approximate the intratemporal equilibrium predictions better than either market-maker model. Namely, the baseline model has an R^2 of .9849 is nearer to perfect fit (i.e., $R^2 = 1$) than either the single or dual market-maker models by about 10^{-2} and 10^{-1}, respectively. The respective RMSE for the baseline model (.5167) is nearly half the RMSE for the single market-maker and a little less than a third the RMSE for the dual market-maker. Thus indicating that the baseline trade prices converge consistently closer to the theoretical predictions than either of the market-maker models.

The measures of fitness of observed simulated outcomes with respect to the theoretically predicted intertemporal equilibrium show the baseline model outperforming the single market-maker model while being outperformed by the dual market-maker model. The baseline model has an R^2 of .8429 with a RMSE of

[17]The coefficient of determination, or R^2, describes the goodness-of-fit between a model and observed data. It is the proportion of the variance in the dependent variable that is predictable from the independent variable.

(1.664) which is a closer approximation than the single market-maker version with an R^2 of .6995 and a RMSE of (2.067). In contrast, the dual market-maker has an R^2 of .8903 and a RMSE of (1.725).

This apparent inconsistency in performance is an artifact of the formulation of the intertemporal equilibrium price. Namely, the intertemporal equilibrium price is formulated as a one period midpoint value forecast. Meanwhile, a market-maker's forecast is an interpolation function of the order of the number of periods of foresight (i.e., O^{τ}). So the larger the foresight range, the greater the discrepancy between the dynamically forecasted price approximation and the theoretical predicted series of intertemporal equilibrium prices.

Comparing coefficients of determination across each market-maker model version shows an effect of market-maker participation on the fitness of the series of trade contract prices with respect to predicted equilibrium prices. In both versions the series of trade prices where neither contracting party was a market-maker shows an improvement in fitness with respect to the intratemporal equilibrium price and a decline with respect to intertemporal equilibrium prices. Similarly, in both versions the series of trade prices where one of the contracting parties was a market-maker shows a decline in fitness with respect intratemporal equilibrium prices. While this series also shows a slight decline in fitness with respect to intertemporal equilibrium prices, it is nearly negligible by comparison. In the single market-maker DA model the series of trade prices where one of the contracting parties is a market-maker has an R^2 of .6926 and a RMSE of (2.124). In the dual market-maker DA model the series of trade prices where one of the contracting parties is a market-maker has an R^2 of .8896 and a RMSE of (1.631). Both cases show a decrease from the R^2 of all trade contract prices of 10^{-3} and only the single market-maker series has a slightly larger deviation.

Figure 7 shows the corresponding series of gains from trade across 10 periods of 12 simulated realities for each version of the DA market model. Using the same graphical representation as in Figures 2, 3 and 5, for each panel the boxplot provides a visually discernible distribution of the gains from trade for every period over all realities. The green line plot represents the set of theoretically predicted competitive equilibrium economic surplus for every period. The red line plot represents the theoretically predicted intertemporal equilibrium economic surplus – corresponding to the intertemporal equilibrium prices. For the second and third panel, the area between the orange lines represents the set of arbitrage gains from trade due to market-maker participation for every period over all realities[18].

[18] These arbitrage gains from trade are a convex combinations of every potential permutation of intratemporal and intertemporal economic surpluses given the possibility of market-maker participation.

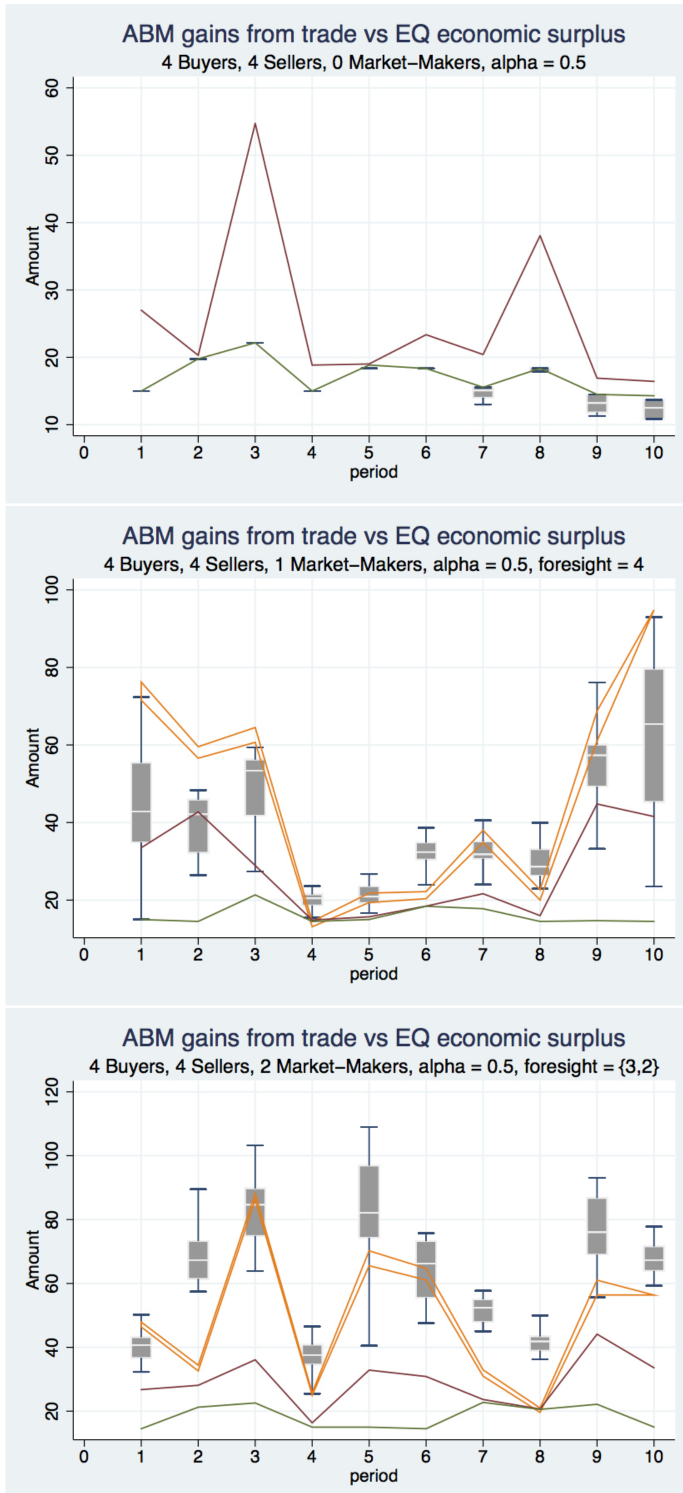


Figure 7: Simulated DA gains from trade vs theoretically predicted economic surplus

The top panel shows the distribution of the gains from trade for every period of the baseline model closely converging towards the intratemporal equilibrium economic surplus. For seven out of ten periods the set of gains from trade collapse almost exactly to their corresponding intratemporal equilibrium predictions.

By comparison, only the distribution of the gains from trade for periods two and five approximate the corresponding intertemporal equilibrium economic surplus – where the intratemporal and intertemporal equilibrium predictions are almost or exactly the same.

The addition of a single market-maker to the baseline model increases gains from trade above the levels predicted by intratemporal and intertemporal equilibrium. The middle panel shows the bulk of the series of gains from trade exceed both the intratemporal and intertemporal equilibrium predictions for nine out of ten periods. In period two the measures of central tendency converge to the intertemporal equilibrium and the bottom half of the distribution of gains from trade lies between the intertemporal and intratemporal equilibrium. By contrast, the bulk of the series of gains from trade fall below the predicted arbitrage gains from trade due to market-maker participation for half of the periods. For periods four to eight the bulk of the series of gains from trade in the simulated DA market either exceed or match the predicted arbitrage gains from trade due to market-maker participation – where the series of arbitrage gains from trade approximate the predicted intertemporal equilibrium surplus.

The bottom panel shows that the addition of a second market-maker increases the gains from trade above all three series of predicted outcomes. Only the first period shows the bulk of the series of the gains from trade in the simulated DA market fall below the predicted arbitrage gains from trade. In periods three and six the measures of central tendency of the series of simulated outcomes converge within the range of predicted arbitrage gains from trade such that half of the distribution falls below. For the rest of the periods the bulk of the series of simulated outcomes exceed the predicted arbitrage gains.

	Mean market efficiency							
	Baseline		Single Market-Maker			Dual Market-Maker		
Period	intra	inter	intra	inter	arbitrage	intra	inter	arbitrage
1	.9833	.5463	2.962	1.326	.6071	2.778	1.506	.8583
2	.9978	.9724	2.719	.9224	.6854	4.539	2.423	2.044
3	1.00	.4049	2.299	1.696	.7923	4.071	2.312	.9628
4	.9833	.7825	1.375	1.342	1.472	2.497	2.293	1.481
5	.9624	.9543	1.425	1.362	1.056	5.462	2.493	1.216
6	.9926	.7800	1.752	1.748	1.542	3.107	2.064	1.017
7	.9458	.7227	1.811	1.487	.8999	3.432	2.176	1.621
8	.9841	.4749	2.043	1.853	1.420	2.338	2.032	2.082
9	.9083	.7788	3.670	1.204	.8526	5.104	1.735	1.314
10	.8611	.7494	4.283	1.495	.6548	4.497	2.009	1.197

Table 7: Mean allocative efficiency of simulated DA market gains from trade with respect to predicted outcomes

Table 7 outlines the measures of allocative efficiency as defined by Smith (1962)[19] for the outcomes of each DA market simulation model with respect to the predicted levels. The table shows that the gains from trade in the baseline DA market model nearly reach full efficiency with respect to the intratemporal equilibrium predictions. The average mean intratemporal efficiency of the baseline simulation over 10 market periods is .9621, or 96.21 percent. Dismissing the results for the last two periods due to boundary issues, the truncated average over the first eight periods increases the mean efficiency to 98.12 percent. These baseline results are nearly indistinguishable from the efficiency levels reported by Gode and Sunder (1993). By comparison, the mean efficiency of the baseline model with respect to the intertemporal equilibrium is .7166, or 71.66 percent. Only periods two and five reach intertemporal market allocative efficiency levels similar to those reported by Miller et al. (1977).

The levels of intratemporal allocative efficiency for the single market-maker model show its series of gains from trade clearly exceed the equilibrium predictions. The average mean intratemporal efficiency of this simulated DA market over 10 periods is 2.43, or 243 percent. Similarly, the levels of intertemporal allocative efficiency show that with the exception of period two the series of gains from trade exceed equilibrium predictions. The average mean intertemporal efficiency of this simulation's gains from trade is 1.44, or 144 percent, which far exceeds the average allocative efficiency of 96.3 percent reported in Miller et al. (1977) speculative market. By contrast, the simulated DA market gains from trade reach full efficiency with respect to the arbitrage gains from trade due to market-maker participation for only four out of the 10 periods. The average mean efficiency of the remaining six periods is .7486, or 74.86 percent.

[19] The allocative efficiency is the sum total of the profits made in the DA simulated market period divided by the theoretically predicted total maximum profit.

When pooled together, the series of gains from trade over all 10 periods reach 99.82 percent allocative efficiency with respect to the arbitrage gains from trade.

The levels allocative efficiency for the dual market-maker model show its series of gains from trade far exceed both the intratemporal and intertemporal equilibrium predictions. The average mean intratemporal and intertemporal efficiencies of this simulated DA market over 10 periods are 3.78 and 2.10 – 378 and 210 percent – respectively. These results exceed the levels in Gode and Sunder (1993) as well as Miller et al. (1977). In contrast to the previous versions of the simulated DA market, the bulk of the series of gains from trade for 8 out of 10 periods surpass the levels of full efficiency with respect to the arbitrage gains from trade. The average mean arbitrage efficiency over 10 periods is 1.37, or 137 percent.

Figure 8 shows the series of gains from trade for the single market-maker model separated by market-maker participation for each market period over 12 simulated realities. The blue scatter plots depict the series of gains from trade where neither contracting party was a market-maker. The red scatter plots depict the series of gains from trade where one of the contracting parties was a market-maker. For each period's scatter plot, the blue and red vertical lines mark the intratemporal and intertemporal equilibrium surplus, respectively.

For periods one, two, three, nine and 10 the gains from trade made where one of the contracting parties was a market-maker are consistently larger than those made where neither was a market-maker. For the bulk of the series in periods four and five this trend is reversed. From periods six to eight the bulk of both series are indistinguishable.

Except for period 10 the bulk of the series of gains from trade where neither contracting party was a market-maker cluster near the intratemporal equilibrium predictions. Except for periods four and five, the bulk of the series of gains from trade where one of the contracting parties was a market-maker cluster about or above the intratemporal predictions. Almost all the series where neither contracting party was a market-maker cluster noticeably below intertemporal equilibrium levels – except for periods four to six, which are closer approximations. For periods one, three, eight, nine and ten the bulk of the series where one of the contracting parties was market-maker cluster above intertemporal equilibrium levels. In periods six and seven the series scatter near the equilibrium level. For periods two, four and five the series cluster noticeably below intertemporal levels.

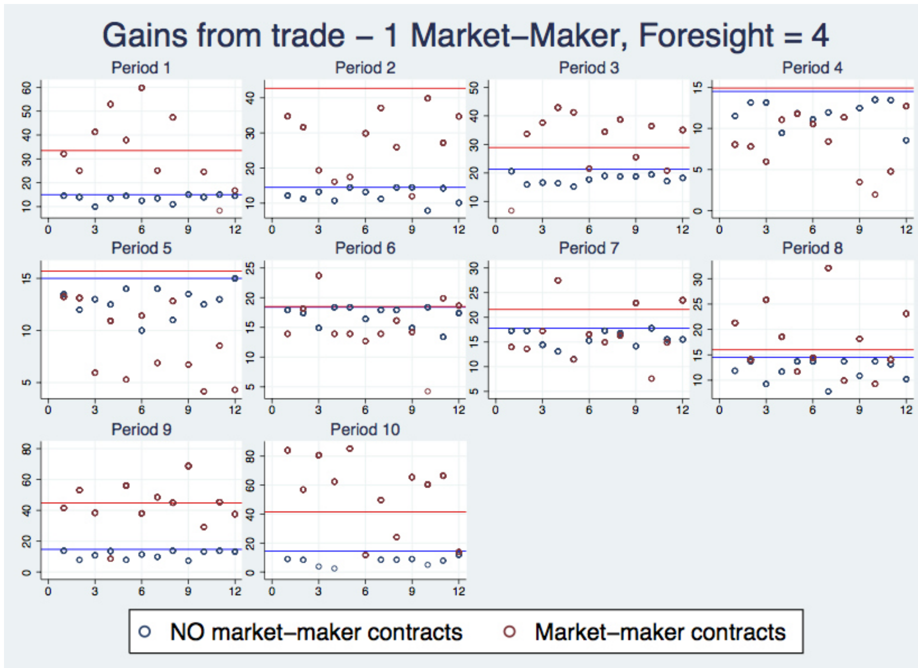


Figure 8: Single market-maker DA market gains from trade by market-maker participation in transactions

	Single market-maker mean efficiency					
	Contracts w/o MM			Contracts w/ MM		
Period	intra	inter	arbitrage	intra	inter	arbitrage
1	.9082	.4066	.1865	2.728	1.221	.5602
2	.8670	.2942	.2186	2.062	.6995	.5201
3	.8436	.6223	.2904	1.634	1.205	.5626
4	.8268	.8067	.8852	.6349	.6195	.6797
5	.8619	.8241	.6394	.6544	.6257	.4854
6	.9262	.9242	.8148	.9145	.9125	.8044
7	.8776	.7207	.4356	1.030	.8462	.5114
8	.8392	.7610	.5845	1.351	1.225	.9411
9	.8159	.2678	.1893	3.169	1.039	.7350
10	.6056	.2114	.0926	4.399	1.536	.6727

Table 8: Single market-maker allocative efficiency by market-maker participation

Table 8 breaks down the measures of allocative efficiency for the single market-maker DA model by whether one of the contracting parties was a market-maker or not. The average mean intratemporal efficiency of the series of gains from trade where neither contracting party was a market-maker is .8372, or 83.72 percent. Setting aside period ten, this series of gains from trade range between 81.59 and 92.62 percent of the intratemporal allocative efficiency. By comparison, the average mean intertemporal efficiency of the same series of gains from trade is .5839, or 58.39 percent. Excluding periods four, five an six, the series of gains from trade range between 21.14 and 76.10 percent of the intertemporal allocative efficiency. Coincidently, the average mean arbitrage efficiency for the series is .4336, or 43.36 percent, and the series range between 9.26 and 58.48 percent when periods four, five and six are excluded.

For the series of gains from trade where one of the parties was a market-maker seven out of ten periods exceed full intratemporal allocative efficiency. The average mean intratemporal efficiency of the series is 1.85, or 185 percent. The mean intratemporal efficiency levels for periods four, five and six are 61.59, 62.57 and 91.45 percent, respectively – the rest of the periods exceed full intratemporal efficiency. The average mean intertemporal efficiency of the series of gains from trade is .9931, or 99.31 percent. For periods two, four, five, six and seven the levels of mean intertemporal efficiency range between 61.95 and 91.25 percent – the rest of the periods exceed full intertemporal efficiency. The levels of arbitrage efficiency of the series range between 48.54 and 94.11 percent with an average mean efficiency of .6473.

Figure 9 shows the series of gains from trade for the dual market-maker model separated by market-maker participation for each market period over 12 simulated realities. The blue scatter plots depict the series where neither contracting party was a market-maker. The red scatter plots depict the series where one of the contracting parties was a market-maker. For each period's scatter plot, the blue and red vertical lines mark the intratemporal and intertemporal equilibrium surplus, respectively.

In contrast to Figure 8, the gains from trade where one of the contracting parties was a market-maker are consistently larger than those where neither contracting party was a market-maker for all periods.

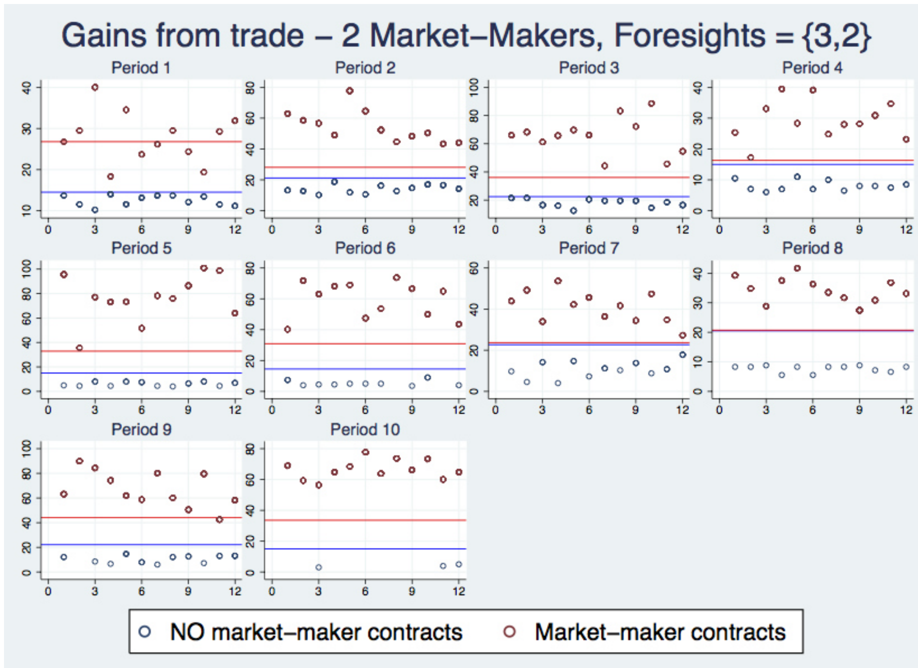


Figure 9: Dual market-maker DA market gains from trade by market-maker participation in transactions

In keeping with the analysis of the series of trade prices for the dual market-maker DA model, the series of gains from trade exhibits an inordinate portion of contracts where one of the parties was a market-maker. The portion of contracts where neither party is a market-maker rapidly dwindles after period four. Coincidently, all the series where neither contracting party was a market-maker cluster well below the intratemporal equilibrium levels for periods 4 through 10. For periods 1 through 3, the series scatter closer around the predicted intratemporal equilibrium levels. By contrast, all the series where one of the contracting parties was a market-maker consistently exceed both the intratemporal and intertemporal equilibrium predictions for all 10 periods.

	Dual market-maker mean efficiency					
	Contracts w/o MM			Contracts w/ MM		
Period	intra	inter	arbitrage	intra	inter	arbitrage
1	.8676	.4703	.2680	2.008	1.0889	.6205
2	.6868	.5194	.4387	2.615	1.978	1.670
3	.8103	.5068	.2110	3.017	1.887	.7856
4	.5568	.5111	.3303	2.101	1.929	1.246
5	.4333	.1978	.0964	5.409	2.469	1.203
6	.3917	.1840	.0908	4.270	2.006	.9904
7	.5257	.5055	.3767	1.857	1.786	1.331
8	.3728	.3674	.3801	1.695	1.671	1.729
9	.5145	.2584	.1955	3.180	1.598	1.209
10	.2663	.1191	.2459	4.465	1.997	1.190

Table 9: Dual market-maker allocative efficiencies by market-maker participation

Table 9 shows the measures of allocative efficiency for the dual market-maker broken down by whether one of the contracting parties was a market-maker or not. Apart from the mean intratemporal efficiencies for periods 1 through 3, the gains from trade where neither contracting party was a market-maker reach 55.68 percent or lower of any efficiency measure. The average mean intratemporal, intertemporal and arbitrage efficiencies for the series of contracts where neither party was a market-maker are .5426, .3639 and .2459 – 54.26, 36.39 and 24.59 percent – respectively.

Consistent with the graphical results, both the mean intratemporal and intertemporal efficiencies for the gains from trade where one of the contracting parties was a market-maker reach levels far exceeding their respective equilibrium predictions. The mean intratemporal efficiency peaks at 541 percent in period 5 and drops as low as 169.5 percent in period 8. The mean intertemporal efficiency peaks at 247 percent in period 5 and drops as low as 109 percent in period 1. The average mean intratemporal and intertemporal efficiencies for the series of contracts where one of the parties was a market-maker are 3.062 and 1.841 – 306.2 and 184.1 percent – respectively. By comparison, except for periods one and three the gains from trade either meet or exceed full arbitrage efficiency levels.

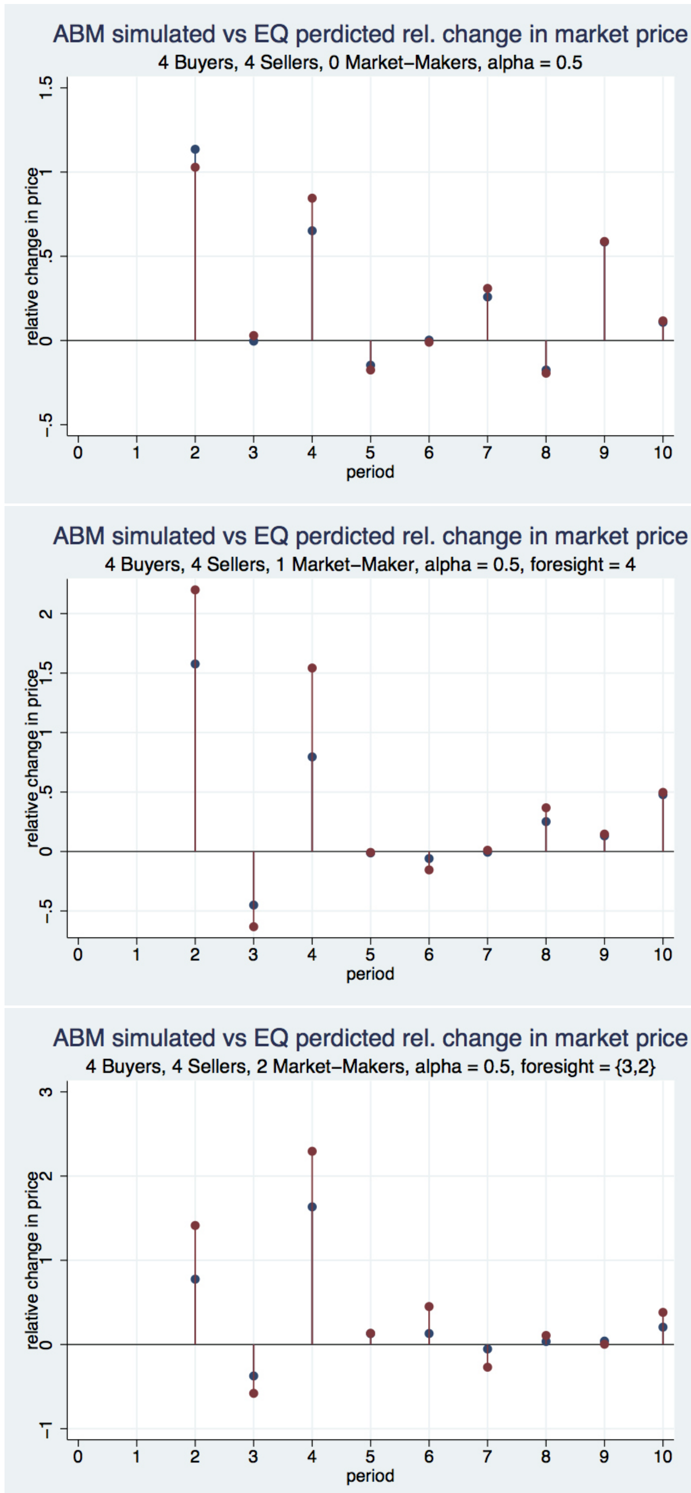


Figure 10: Relative change in market prices between periods: simulated vs predicted

The mean arbitrage efficiency peaks at 173 percent in period five and drops as low as 62 percent in the first period. The average mean arbitrage efficiency for contracts where one of the parties was a market-maker is 1.198.

Figure 10 shows the relative change in trade contract prices and competitive equilibrium prices for each version of the simulated DA market. The relative change in price for period t is the absolute value of the difference in price between $t$and $t-1$ divided by the price in period $t-1$, $\frac{P_t - P_{t-1}}{P_{t-1}}$. The blue dropped-line series represents the relative change in the median trade contract price in each simulated market period over 12 realities. The red dropped-line series represents the relative change in the theoretically predicted competitive equilibrium prices. The top panel shows the relative changes in trade contract prices closely approximate the relative changes in competitive equilibrium prices in the baseline DA market. The middle and top panels show relative changes in trade contract prices consistently smaller than relative changes in competitive equilibrium prices by similar orders of magnitude for the single and dual market-maker DA markets, respectively.

	absolute value of relative change in price, $\lvert\frac{\Delta P_{t-1}}{P_{t-1}}\rvert$					
	Baseline		Single Market-Maker		Dual Market-Maker	
Period	simulated	predicted	simulated	predicted	simulated	predicted
1	–	–	–	–	–	–
2	1.135	1.028	1.576	2.199	.7759	1.414
3	.0037	.0296	.4503	.6326	.3737	.5796
4	.6515	.8452	.7905	1.542	1.634	2.294
5	.1463	.1751	.0117	.0078	.1301	.1325
6	.0025	.0098	.0592	.1550	.1307	.4507
7	.2588	.3095	.0059	.0103	.0530	.2699
8	.1749	.1948	.2515	.3675	.0351	.1074
9	.5848	.5880	.1309	.1465	.0411	.0030
10	.1079	.1165	.4777	.4968	.2058	.3812
μ	.3407	.3663	.4176	.6176	.3755	.6258
cv	1.109	1.007	1.216	1.229	1.404	1.205

Table 10: Absolute value of relative change in market price between periods

Table 10 outlines the absolute value of relative price change for the series of trade contracts and competitive equilibrium predictions for each version of the simulated DA market. Other than period two, the relative change in prices in trade contracts for the baseline DA market simulation are smaller than the predicted competitive equilibrium prices. For period two the relative change in trade contract prices is .107 bigger than the relative change in competitive equilibrium prices. By contrast, the relative change in trade contract prices in period 4 is .1936 smaller than the relative change in competitive equilibrium prices. For the rest of the periods the relative changes in trade contract prices are with 10^{-2} to 10^{-3} smaller than the relative changes in competitive equilibrium prices. The average of the absolute values of relative price changes in trade contracts for the baseline DA market is .3407, while it is .3663 for the predicted equilibrium prices.

The absolute values for both market-maker DA models exhibit a pronounced reduction in relative change in the simulations' trade contract prices with respect to their corresponding relative changes in competitive equilibrium prices. Namely, the relative changes in trade contract prices are noticeably smaller than the corresponding relative changes predicted in competitive equilibrium for periods with a steep change with respect to the previous period's equilibrium price. For periods with a very slight change with respect to the previous period's equilibrium price the differences in relative change are nearly negligible. For example, period 5 in the single market-maker DA simulation with an equilibrium price decrease of .0618 with respect to period 4 shows a relative change in trade contract prices slightly bigger than the relative change in competitive equilibrium prices – similarly for period 9 in the dual market-maker simulation.

Overall, the introduction of a market-maker exhibits an attenuating effect on the relative change in trade contract prices. The average absolute value of relative change in prices for trade contracts and competitive equilibrium predictions in the single market-maker DA model are .4176 and .6176 – 41.67 and 61.76 percent – respectively. Similarly, the average absolute value of relative change in prices for trade contracts and competitive equilibrium predictions in the dual market-maker DA model are .3755 and .6258 –37.55 and 62.58 percent – respectively.

Concluding Remarks

Taking the intertemporal equilibrium measurement proposed by Miller et al. (1977), the introduction of speculative intermediacy into a competitive market with random unknown shifts in demand and supply yields gains from trade almost uniformly exceeding full intertemporal market efficiency. Absent a market-maker (speculator), boundedly rational (non-optimizers) buyers and sellers in a multilateral exchange process with decentralized information are able to reach near full efficiency levels within most market periods. Conversely, absent a market-maker most of the market periods reach between 2/5 and 4/5 of full intertemporal efficiency.

The levels of intratemporal and intertemporal market efficiency in trade transactions between buyers and sellers in a market with a single market-maker are qualitatively similar to those in a market with no market-maker. Namely, they exhibit a similar pattern of variation across periods with slightly lower values. In contrast, trade transactions where one of the contracting parties is a market-maker show a disproportionate reduction in intra and intertemporal efficiencies across periods with small differences in intratemporal equilibrium prices. Specifically, periods four to six drop below full intra and intertemporal efficiency while period seven barely reaches full intratemporal efficiency but drops below full intertemporal efficiency. For the market with two market-makers the low volume of transactions between buyers and sellers show lower values but a similar pattern of variation to the intra and intertemporal levels of efficiency of trade between buyers and sellers in the previous markets. The inordinate number of transactions where one of the contracting parties is a market-maker show reductions in intra and intertemporal efficiencies qualitatively similar to those for a market with a single market-maker. While all periods surpass full intra and intertemporal efficiency levels, periods four, seven and eight drop near or below their respective averages.

The predicted arbitrage gains from trade are measurements to contrast the intra and intertemporal autarkic market equilibrium outcomes with the additional potential profits made possible by speculative intermediation. As such, they provide a linear combination of the average gains from every possible trade of intra and intertemporal equilibrium inframarginal units given market-makers' forecasts. Both markets with speculative intermediacy are able to reach full arbitrage efficiency where relative changes in equilibrium prices between periods are small. For the rest of periods the market with a single market-maker reaches between $4/7$ and $6/7$ of full arbitrage efficiency, where the vast majority is attributable to transactions with the market-maker. For the market with one additional market-maker, the bulk of the profits from trade in the remaining periods exceed full arbitrage efficiency – almost entirely attributable to the overwhelming portion of trades where one of the contracting parties was a market-maker.

To the extent that trade prices in the baseline model closely approximate intratemporal equilibrium predictions, the introduction of speculative intermediacy shows a tendency to increase the volatility of market-clearing prices within each market period. In spite of acyclical shifts in demand and supply between market periods, contract prices in the baseline simulation exhibit a rate of convergence toward intratemporal equilibrium similar to those in human experiments. According to the coefficients of determination in Table 6, adding a market-maker increases the rate of dispersion for each market period's set of trade contract prices and reduces goodness-of-fit with the respective competitive equilibrium predictions. Similarly, within markets with speculative intermediacy the series of trade prices where one of the contracting parties was a market-maker shows greater dispersion and reduced goodness-of-fit to intratemporal equilibrium than the series of trade prices where neither contracting party was a market-maker. For either market with speculative intermediacy, the series of trade prices where neither contracting party was a market-maker show less dispersion and closer convergence to intratemporal equilibrium prices than the series of ALL trade contract prices – qualitatively similar to rates in the market without market-makers. Coincidently, the series of trade prices where one of the contracting parties is a market-maker show more dispersion and sparser convergence to intratemporal equilibrium prices than the corresponding series of ALL trade contract prices – qualitatively similar to rates in the markets with market-makers.

Insofar as the intertemporal equilibrium measurement adapted from Miller et al. (1977) is a linear midpoint approximation between one period and the next, the sparser convergence and greater dispersion of trade prices in the DA market with a single market-maker with respect to the baseline market is an artifact of the market-maker's forecasts based on a 4^{th} degree polynomial approximation. By contrast, the DA market with dual market-makers using quadratic and cubic polynomial approximations to forecast future prices show closer convergence but a greater dispersion of trade prices. For either market with speculative intermediacy, the series of trade prices where neither contracting party is a market-maker shows more dispersion and sparser convergence to intertemporal equilibrium than the corresponding series of ALL trade contract prices. Both markets with speculative intermediacy show nearly identical rates of convergence to intertemporal equilibrium prices between the series of ALL trade contract prices and the series of trade prices where one of the contract parties was a market-maker. For the DA market with a single market-maker, the series of trade prices where one of the contracting parties is a market-maker shows greater dispersion than the set of ALL trade contract prices. Meanwhile the dual market-maker case shows a decrease in dispersion.

A more revealing measure of the effect of speculative intermediacy on a DA market with acyclical shifts in supply and demand is the relative change in trade contract prices compared to the relative change in intratemporal equilibrium prices. Absent a market-maker, the difference in the relative changes in price between the series of trades in each period of the DA market and their respective competitive equilibrium predictions is almost negligible. By contrast, both DA markets with speculative intermediacy show a significant attenuation in the relative change in trade contract prices with respect to the relative change in predicted equilibrium prices across periods. In other words, the introduction of speculative intermediacy reduces the volatility of the series of prevalent market-clearing prices across trading periods in DA market with acyclical shifts in supply and demand.

Far from constituting a definitive microeconomic framework of intermediation as postulated by Spulber (1998), this study provides a simple speculative intermediacy model of decentralized intertemporal price formation in competitive markets with unknown acyclical shifts in supply and demand. The simulations' findings show that every additional market-maker increases consumer and producer surplus and reduces the volatility of market-clearing prices across periods at the cost of increased volatility of trade contract prices within periods. Factoring issues of difference in polynomial degrees between forecasted market-clearing prices and intertemporal equilibrium prices, these findings show *boundedly rational* market-makers are able to reach competitive intertemporal outcomes without *perfect foreknowledge* of a market with acyclical random shifts in supply and demand. In other words, these findings suggest that neither rationality nor complete information are necessary conditions to reach intertemporal allocative efficiency in a speculative market. While further improvements to intertemporal measurements are needed, these findings indicate that acyclical DA markets with unknown shifts in demand and supply can reach intertemporal competitive equilibrium under similar conditions that yield competitive equilibrium in decentralized trade among non-optimizers in stationary markets as shown by Gode and Sunder (1993).

References

[1] Becker, G.S., (1962) "Irrational Behavior and Economic Theory." The Journal of Political Economy, **70**: 1-13

[2] Crockett, S., Spear, S. and S. Sunder (2008) "Learning Competitive Equilibrium." Journal of Mathematical Economics, **44**: 651-671

[3] Crockett, S. (2008) "Learning competitive equilibrium in laboratory exchange economies." Economic Theory, **34**(1): 157-180

[4] Camerer, C. F. (2003) *Behavioral Game Theory.* Princeton University Press, Princeton

[5] Duffy, J. and M. U. Ünver (2006) "Asset price bubbles and crashes with near-zero-intelligence traders." Economic Theory, **27**: 537-563

[6] Duffy, J. (2006) "Agent-Based Models and Human Subject Experiments." *Handbook of Computational Economics*, Vol. 2, Elsevier, 950-1005

[7] Epstein, J.M. and R. Axtell(1996) *Growing Artificial Societies*. Brookings Institution Press, Washington, D.C.

[8] Feltovich, N. (2000) "Reinforcement-Based vs. Belief-Based Learning Models in Experimental Asymmetric Information Games." Econometrica, **68**: 605-641

[9] Friedman, D. (1984) "On the Efficiency of Experimental Double Auction Markets." American Economic Review, **74**(1): 60-72

[10] Gode, S. and S. Sunder (1993) "Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality." The Journal of Political Economy, **101**(1): 119-137

[11] Gode, S. and S. Sunder (1997) "What Makes Markets Allocationally Efficient?" The Quarterly Journal of Economics, **112**(2): 603-630

[12] Miller, R. M., Plott, C. R. and V. L. Smith (1977) "Intertemporal Competitive Equilibrium: An Empirical Study of Speculation." The Quarterly Journal of Economics, **91**(4): 599-624

[13] Munger, M. C. (2011) "Euvoluntary or Not, Exchange is Just." Social Philosophy and Policy, **28**(2): 206-235

[14] Plott, C. and S. Sunder (1982) "Efficiency of Experimental Security Markets with Insider Information: An Application of Rational-Expectations Models.", Journal of Political Economy, **90**(4): 663-698

[15] Plott, C. (1982) "Industrial Organization Theory and Experimental Economics." Journal of Economic Literature, **20**(4): 1485-1572

[16] Plott, C. (1986) "Rational Choice in Experimental Markets." The Journal of Business, **59**(4): S301-S327

[17] Plott, C. and S. Sunder (1988) "Rational Expectations and the Aggregation of Diverse Information in Laboratory Security Markets." Econometrica, **56**(5): 1085-1118

[18] Rust, J. and G. Hall (2003) "Middlemen versus market makers: A theory of competitive exchange." The Journal of Political Economy, **111**(2): 353-403

[19] Simon, H. A. (1996) *The Science of the Artificial*. MIT Press. Cambridge, MA.

[20] Smith, V. L. (1962) "An Experimental Study of Competitive Market Behavior." The Journal of Political Economy, **70**(2): 111-137

[21] Smith, V. L. (1976) "Experimental Economics: Induced Value Theory." American Economic Review, **66**: 274-279

[22] Smith, V. L., Williams, A. W., Bratton, W. K. and M. C. Vannoni (1982) "Competitive Market Institutions: Double Auction vs. Sealed Bid-Offer Auctions" American Economic Review, **72**(1): 58-77

[23] Smith, V. L. (1982) "Microeconomic Systems as an Experimental Science." American Economic Review, **72**(5): 923-955

[24] Spulber, D. F. (1996) "Market Microstructure and Intermediation." Journal of Economic Perspectives, **10**(3): 135-152

[25] Spulber, D. F. (1998) *The Market Makers: How Leading Companies Create and Win Markets*. McGraw-Hill

[26] Spulber, D. F. (1999) *Market Microstructures: Intermediaries and the Theory of the Firm*. Cambridge University Press, Cambridge.

[27] Sunder, S. (2003) "Markets as Artifacts: Aggregate Efficiency from Zero Intelligence Traders." Yale ICF Working Paper.

[28] Schelling, T. C. (1978) *Micromotives and Macrobehavior*. W.W. Norton & Company

[29] Taber, C. S. and R. J. Timpone (1996) *Computational Modeling*. Sage Publications, Inc.

[30] Tuner, S. (1836) *The History of the Anglo-Saxons*. Longman, Rees, Orme, Brown, Green, and Longman. London.

Appendix A: Software documentation

This section is intended as an overview of the anatomy and physiology (i.e., structure and functionality) of the simulation. Its main purpose is to provide the reader with a better understanding of the different components and how they relate to one another. There are three distinct interacting modules that comprise the overall simulation program.

First, a *value generator module* takes an endogenously determined microstructure of the trade environment. This module consists of a single MATLAB microsimulation program. It reads in a text file containing a general description of the economic agents, resources, schedules of valuations, and constraints. Then it writes out a series of vector structures that assign the initial market environment attributes: resource allocation, induced valuations, etc.

Second, a *Double Auction (DA) simulation module* generates the behavioral model. Artificial agents interact given induced valuations and market rules (i.e., institutional settings). This *Agent Based Model (ABM)* consists of eight interacting submodules programmed in JAVA.

Third, *statistical analysis module* takes in the ABM simulated data to measure the effect of an intermediary (or market-maker). Specifically, it measures and compares the outcomes of a baseline version without market-makers and a version with market-makers using *STATA*. In addition, it measures both ABM versions with the theoretically predicted outcomes from the microsimulation.

Value Generator Module

The value generator is a discrete event stochastic microsimulation written in MATLAB. It reads in a text file with fixed parameters representing general market attributes. These would include the number of trading units, buyers, sellers, market-makers, and time periods, as well as corresponding values and costs.

The microsimulation uses this information to generate a baseline supply-and-demand model with buyers and sellers trading across periods. It replicates this model with the participation of market-makers. This traces the theoretical equilibrium price and quantity for both settings, with and without market makers. Price volatility and market efficiency are compared between these scenarios. Finally, it writes out all this information in vector structures. These are read by the ABM module that assigns initial attributes to DA market and its agents.

Global static parameters input file

The input file contains the global static parameters, which are the *'building blocks'* of the microsimulation. It is a simple flat text file[20] made up of rows of tabulated numerical information. Each row represents a series of records separated by line-breaks. And each record consists of a series of non-identical fields separated by the tab character. It can be directly typed with a text editor or created through a JAVA user interface. The following picture shows one example of an input file.

inputFile.txt

```
33.3
4
4
10
2
12	12
6.0	5.45	4.9	4.35	3.8	3.25	2.7	2.15	1.6	1.05	1.0	1.0
6.0	5.5	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0	1.0
2
12	12
1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.0
1.0	1.55	2.1	2.65	3.2	3.75	4.3	4.85	5.4	5.95	6.0	6.0
2
1
10	4	2
100.0
1000.0
2
3	2
0.5
0.5
```

This text file is a list of vectors of number values. Each vector contains one or more number elements of either real or integer values. As it is visually discernible, all elements in a record vector are the same type of number. The elements in the list displaying a digital fraction are real numbers – whole numbers are integers.

The first row is the *'seed'* for MATLAB's random number generator. The second, third, fourth and fifth rows are the number of buyers, number of sellers,

[20]A flat file is a data file containing records with no structured relationships. It can be a plain text file with one record or more per line. Each field within a record is separated by a delimiter consisting of either a comma, semi-colon, or tab space. For the purposes of this study, a *flat text file* will refer to the latter.

number of periods and number of demand curves, respectively. The sixth row is a vector of the number of elements demanded for each demand curve. The seventh and eight rows are vectors of the unit values in a demand curve.

The ninth and tenth rows are the number of supply curves and the corresponding vector of their respective number of elements supplied. The eleventh and twelfth rows are vectors of the unit costs in a supply curve.

The thirteen row signals the *allocation method*. If the number is '1' the allocation of units is uniform, and if it's '2' the allocation is random. The fourteenth row is the number of trends that randomly shape the supply and demand curves. The fifteenth row is a vector of the number of trend periods, the upper and lower limits of the trends.

The sixteenth and seventeenth rows are the overspending penalty and budget endowment, respectively.

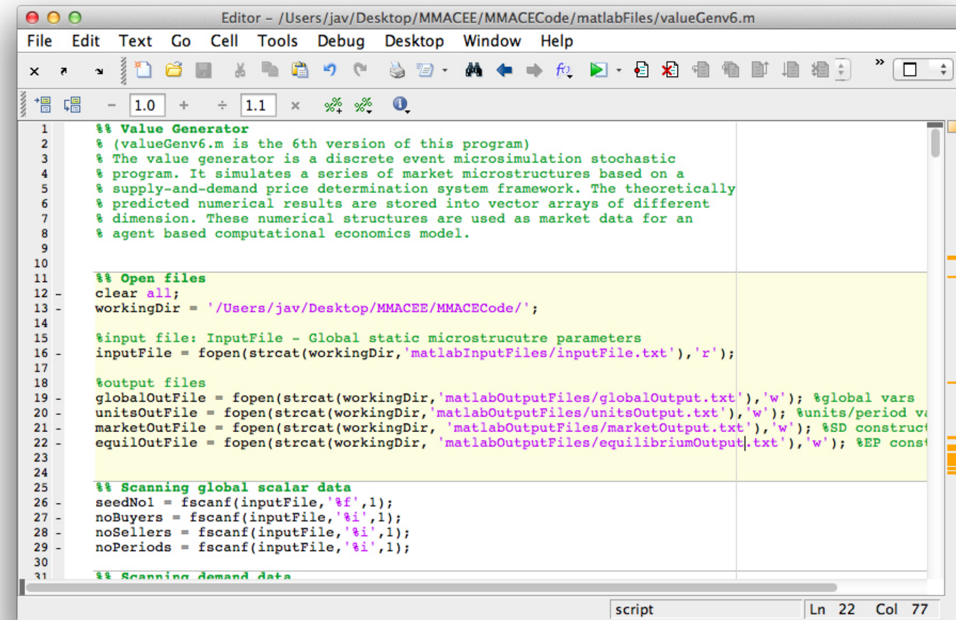
The eighteenth and nineteenth row are the number of market-makers and the corresponding vector of their respective number of periods of foresight.

The twentieth and twentieth-first rows are the anchoring weights for the buyers and sellers, respectively.

This text file of globally static parameters feeds into the MATLAB microsimulation to run and build the market structure. The terms 'unit value' and 'unit cost' are equivalent to willingness to pay (WTP) and willingness to accept (WTA), respectively. The trend variable represents trends in the elasticity of supply and demand, not in prices or quantities.

MATLAB value generator file

The value generator is a numerical simulation program written in MATLAB. It reads in the parameters from the input file and generates a microsimulated series of market structures. It was possible to construct the value generator module in JAVA or any other multi-paradigm, general purpose language (i.e., C, C++, SmallTalk). I opted to build it in MATLAB because it allows for very fast and easy vector building and manipulation.



Upon being initialized, the value generator creates four writeable text files. These are flat text files created to hold the microsimulation's final data structures. The names of the four files are *globalOutput*, *unitsOutput*, *marketOutput*, *equilibriumOutput*.

- *globalOutput* holds the general attributes of the economic agents and the DA market institution: number of periods and agents, maximum values, anchoring weights for convergence to true valuations, and penalties.
- *unitsOutput* file holds the number of per-period units for sale by each seller and number of per-period units desired by each buyer.
- *marketOutput* file holds two dimensional arrays. One contains values for each desired unit by each buyer; the other array has the costs for each unit for sale by each seller throughout the periods.
- *equilibriumOutput* file holds the equilibrium price.

Next, the global parameters are read from the input file. For single-field records this program scans the file for a single entry. Each entry is formatted according to the type of character value (integer, real, string). For multiple-field records one-dimensional arrays were formed to scan and store each field one by one.

After assigning the data to the initial variables, the program constructs six types of vectors to hold the microsimulated data. The data represents a microeconomic structure of supply and demand.

- *value_cost* is a two-dimensional vector with a two-dimensional sub-vector of sellers' costs and buyers' values for each period.
- *equilPrice* and *equilQty* are one-dimensional vectors with the equilibrium prices and quantities calculated for each period.
- *buyersUnits* is a two-dimensional vector with the per-period number of desired units by each buyer.
- *sellerUnits* is a two-dimensional vector with the number of units for sale.
- *SDSet* is a two-dimensional vector with index pairs of the supply and demand vectors used in each period.

Before replicating the double auction trading periods, a Brownian motion function simulates a random walk with a given trend and drift.

At each trading period a new seed number is assigned to a random number generator (RNG). This generator is used in assigning the indexes of the supply and demand curves to be selected. In the first trading period, it simply selects the supply and demand vectors assigned randomly from all the possible choices.

For all other periods the random number generator also finds an equilibrium point where the vectors of decreasing values and increasing costs overlap. The overlap happens between an inframarginal point where value is greater than cost, and a extramarginal point where value is less than or equal to cost. The equilibrium price is the midpoint between a range of prices bound by an upper and a lower limit. The upper limit is the lowest price between the inframarginal value and the extramarginal cost. The lower limit is the highest price between the inframarginal cost and the extramarginal value.

A period trend is set within given upper and lower bounds to reshape and randomly shift the supply and demand curves. If no trend is set, supply and demand shift to a midpoint between the equilibrium prices from the current and previous period. If a trend is set, supply and demand shift to a random-walk point within the established bounds.

After the shift, the random number generator checks whether or not the supply and demand curves still cross. If the vectors of costs and values do not have an overlapping point, it shifts both curves by an arbitrary distance between a random-walk point and the current equilibrium price. Any post-shift negative values and costs vector components get assigned a zero value, and a new maximum value is determined. With the demand and supply curves properly aligned, the units desired or to be sold are assigned either uniformly or randomly.

In a uniform allocation (i.e., allocation value = 1), each buyer and each seller get assigned an equal amount of units. The values and costs are sorted randomly and assigned to each buyer and seller one-by-one. Then the buyers' values get sorted in a descending manner while the sellers' costs get sorted in an ascending manner to resemble the demand and supply schedules.

If the allocation is done randomly (i.e., allocation value = 2), each buyer and seller get one initial unit assigned along with a randomly selected value or cost. The remaining units and their respective values and costs are assigned among the vectors of buyers, sellers, values and costs with a randomly generated index number.

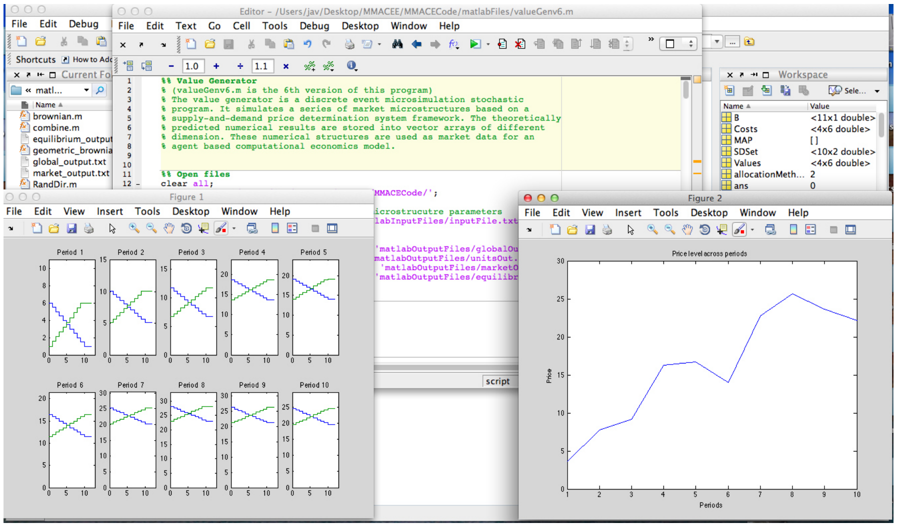
The random allocation of units can create an asymmetry in vector sizes among agents. Buyers and sellers with fewer than the maximum number of units get assigned an additional number of 'non-tradable' units. This is done until all agents of the same type have the same number of units. These non-tradable units get a value of zero for a buyer and a cost greater than the maximum price for the seller. This procedure brings uniformity in units and vector sizes. The vector structures stay symmetric and matrix operations are expedited. Units and values are sorted in the same manner as the uniform allocation of units to resemble demand and supply schedules.

Once all units and values are assigned, the equilibrium quantity is calculated in a manner similar to the equilibrium price. A graphical representation of the current period's supply and demand price determination is then built.

At the end of each period all vector information of the model gets stored in the vector structures created at the beginning of the microsimulation. Values, costs, number of units, and equilibrium price-quantity pairs get added into *values_ costs*, *buyerUnits*, *sellerUnits*, *equilPrice* and *equiQty*.

After repeating this process for all trading periods, the data are written into the corresponding flat text files. All buyers, sellers, and institutional attributes are recorded into the *globalOutput*. The number of units for each buyer and seller get recorded into the *unitsOutput*. All values and costs get recorded into the *marketOutput*. The equilibrium price-quantity pairs get recorded into the *equilibriumOutput*.

Finally, the graphical representation for each period's supply and demand model is plotted together with the graph of price levels across periods.

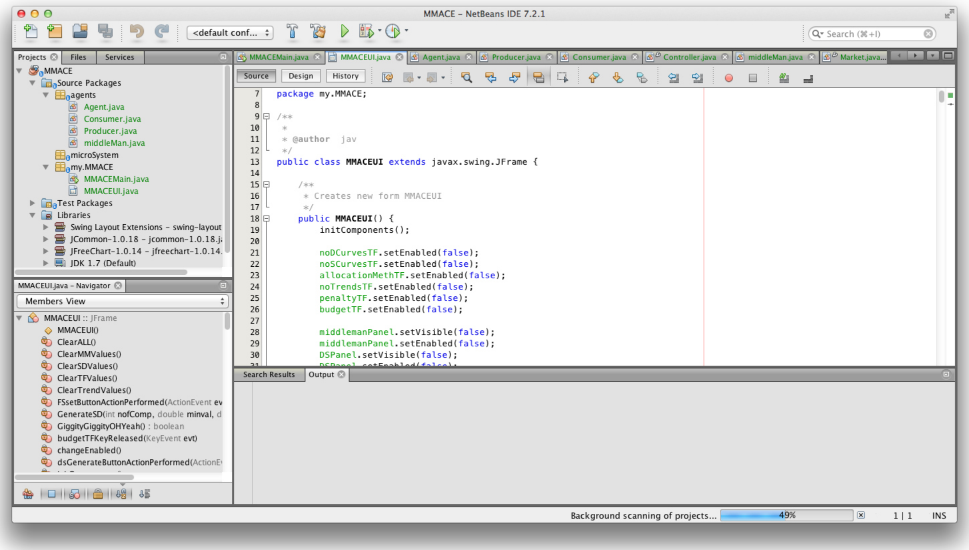


Double Auction Simulation Module

The DA simulation module consists of autonomous agents interacting in an economic environment through a set of institutional rules. The code is in JAVA because it is a general purpose, class-based, object-oriented computer programming language.

The object oriented programming (OOP) allows concepts to be represented as objects with attributes and methods. The OOP attributes are data fields describing objects. Methods are procedures to be carried out by these objects. A class is a self-instantiating, self-referring construct with its own states and behavior.

The interface development environment (IDE) used to edit the JAVA code was Netbeans. Netbeans has the most comprehensive collection of built-in libraries and manuals as well as an easy to use graphical user interface (GUI) toolkit.



The ABM simulation consists of three distinct super-classes, or agent sub-modules. A controller class represents the double auction. It allocates resources and access to exchange according to a set of trading rules. A market class represents the economic environment with the distinguishing attributes of the market. An agent class represents an individual agent interacting in this system. This class has three instances or subclasses: buyer, seller, and market-maker. Each class is an agent type with the corresponding attributes and behavior.

In OOP all classes are instantiated and interact in a main class or construct. In the beginning all the microsimulation data structures are read from the MATLAB output files and loaded into JAVA matrix structures. Then the general attributes of the controller and all the specific attributes of each player are assigned.

Each agent formulates a trading strategy based on a dynamically adjusted learning process. At the beginning of every period, buyers and sellers formulate offers according to the limited information given by their initial attributes. As the number of trades increases buyers and sellers update their offers taking into account the 'going prices' of the trades. Their offer formulation strategies reflect a tradeoff between a running average of trading prices and a converging tendency to reveal their true valuations.

Market-makers are more sophisticated. Their offer formulation strategies reflect the ability to aggregate information across periods to turn an intertemporal profit. Specifically, it reflects their ability to infer future outcomes with some certainty.

In each period agents trade according to DA rules until all units run out or they reach a 'stalemate' in standing offers. At the end of each period, all the information from trades is saved into a series of data structures. When there are no more trading periods, this information is recorded into a comma-separated values (CSV) file for statistical analysis.

Market class

The Market class, or market, is the set of all market attributes. In the beginning of the simulation it retrieves the name and location of the MATLAB output files. Then it opens a file input stream for each corresponding flat file: *globalsFile*, *unitsFile*, *marketConstructFile*, *equilibriumFile*.

The market calls on one method to read and process the data of each input stream.

- The *buildMarketGlobals* method reads the *globalsFile* stream and stores its data in a one-dimensional string list-array.
- The *buildUnitsArray* method reads the *unitsFile* stream and stores its data in a two-dimensional integer value array. The number of rows is the number of buyers and sellers. The number of columns is the number of trading periods. This array stores the number of units for each agent at each trading period.
- The *buildMarketConstruct* method reads the *marketConstructFile* stream and stores its data in a two-dimensional real-value array. The rows correspond to the number of buyers and sellers. The columns correspond to the periods multiplied by the units per agent. It stores each agent's valuations for each unit at each period.
- The *buildEquilibrium* method reads the *equilibriumFile* stream and stores its data in a two-dimensional real-value array. It has two rows; the number of columns equals the number of trading periods. It stores the equilibrium price and quantity for each period.

At each trading period, the controller class calls upon the market to retrieve all period specific data. The list of market method calls is too extensive and thus forgone.

Controller class

The controller is the rule mechanism of this simulated microeconomic system. It assigns property and communication rights to each agent. Before trade begins, it retrieves the set of global variables from the market and instantiates buyer, seller, and market-maker. The controller assigns each agent a specific anchoring weight, type, and index. For each market-maker it additionally assigns an initial endowment, units, and inventory costs.

Throughout the trading periods the controller acts as an automated auctioneer. At the beginning of each period, the *clearingPeriodData* method erases data from previous periods. Then the *buildPeriodData* method reinitializes and constructs the set of microeconomic data structures for the current period.

The *setSeed* method assigns a different seed to each buyer, seller and market-maker to generate random numbers. The *setMinPrice* and *setMaxPrice* methods set the minimum and maximum prices for that trading period. The *setValues* method sets the current period's vector of valuations for each buyer. Similarly the *setCosts* method sets the current period's cost vector for each seller.

The *updateMMUnits* method updates the current period units for the market-makers. The *updateInvCosts*, *updateIntRates*, and *updateMMDelta* methods update the market-makers' inventory costs, interest rates, and learning weight.

The *stoppingRule* method is a cyclic redundancy check. It continuously checks after each offer whether or not a set of three trading conditions are met. The first condition checks whether or not there are any units left to buy. The second condition checks whether or not there are any units left for sale. The third condition checks whether or not the trading offers being posted are converging. When all three conditions are met the trading period continues.

The *callAgentIndex* method emulates a market call for offers. It randomly sets an integer number and selects the agent with a matching index number to place an offer. From here there are four possibilities.

- If the agent has any units left to trade, the controller takes its offer.
- If the agent is a buyer, the *updateBidEvent* method takes in the buyer ID along with its bid. If the entering bid is lower than the standing bid, or highest bid on record, the bid is discarded.
- If the entering bid is higher than the standing bid (but lower than the standing ask) the *setStandingBid* method registers the entering bid as the standing bid. The same is true if the entering bid is the lowest asking price on record.
- If the entering bid is higher than the standing bid and standing ask then a sale contract is made.

A trading price is registered and the standing bid is set to the minimum price while the standing ask is set to the maximum price. Buyers and sellers entering a sale contract decrease their units by one. They set the corresponding valuations out of trading range and register their profits in their respective accounts.

After the trade is made, the *setStandingBid* and *setStandingAsk* methods reset all offers in the market. The process to call for offers is repeated. All trades, trade prices, bids, asks, and standing offers in each period get registered. Calling for offers and trading continues contingent on the *stoppingRule* method conditions being met. Something similar happens for the seller. Verification of whether or not the entering asking price is lower than the standing ask and the standing bid is simply reversed.

If the *callAgentIndex* method 'selects' a market-maker, it is necessary to also determine the type of offer being made. Updating standing bids and asks follows the same process as before. But valuations do not need to be set out of trading range; market-makers formulate offers according to their future expectations.

Once all units have been traded (or offers do not converge) the stopping rule signals the current trading period to stop. The *printPeriodTradesRecord* method prints out to file the record of all market calls, bids, standing bids, asks, standing asks, trade events, trade prices, running average prices, midpoint spreads, and the participating agents. This continues while there are periods to trade.

Agent

The agent is a super-class with a series of attributes and procedures inherent to the buyer, seller, and market-maker agents. It's an abstract agent construct superseded by extensions of itself to its subclasses. Each type of agent extends the agent class by adding or modifying attributes and methods to its own specifications. Upon being instantiated the agent-construct method assigns itself a type, a seed, and an index. The type and index can be used separately or together as identifiers. The seed helps to generate uniformly distributed random numbers for multiple purposes.

The *setType* and *setIndex* methods dynamically change the parameter values of an agent's type and index. The *setRandomNumber* method uses seeding numbers to dynamically change its random number generator. The *getType*, *getIndex* and *getRandomNumber* methods retrieve the type, index and random number.

The main component of the agent's decision making mechanism is dynamic adaptive learning. This enables the agent to change the weigh coefficients of parameters in the formulation of a strategy. A common parameter in the formulation of offering strategies for all agent types is the running average price

– or moving average price. It provides agents with public information of the price trend, reflecting the preferences in the market place. Agents can adjust their offerings to better their chances of making a trade. To this end, the *updateAvgPrice* and *getAvgPrice* methods set and retrieve the running average price, respectively.

Another aspect of this adaptive learning behavior is an agent's tendency to modify its willingness to reveal its true valuation. As more offers are posted and sale contracts realized, the agent increasingly adjusts its willingness to take an offer closer to its true valuation. The *setAdjustedSequence* method sets a sequence by using a global counter, its number of units left, and its previous sequence since its last trade.

The number of units may be greater than or equal to zero; the trade sequence counter is less than or equal to zero. The agent's adjusted sequence is set equal to the sequence counter. The number of units may greater than or equal to zero; the trade sequence counter is greater than zero. The adjusted sequence is equal to a global counter multiplied by an exponential function with a sigmoidal function for an exponent. Otherwise, the adjusted sequence just equals the global counter.

The *getAdjustedSequence* method retrieves this sequence counter and uses it in the formulation of an offering strategy.

Producer

The seller (producer class) is an extension of the agent class. It possesses all the same attributes and methods with modifications specific to its characteristics. Upon being instantiated it gets its type, vector of costs, maximum asking price, anchoring (learning) weight, seed, and index assigned. The *setCosts* and *getCosts* methods set and retrieve the vector of costs. If a seller enters a trade, the *decUnits* method reduces its number of units by one. The *setUnits* method sets the number of units. The *getUnits* method retrieves the number of units the seller has left.

The *getAsk* method formulates an asking price offer given a global counter and running average price. At the beginning of a trading period, the seller formulates an offer by picking an asking price between the maximum price and its true cost for the unit up for trade. As the number of offers increases and the running average price is known, the offering strategy changes. The asking price is set equal to the weighted sum between a willingness to accept converging to its true cost and the running average price.

Consumer

The buyer (consumer class) possesses all the same attributes and methods with modifications specific to its characteristics. Upon being instantiated it gets its type, vector of values, minimum bid, anchoring (learning) weight, penalty, seed, and index. The *setValues* and *getValues* methods set and retrieve the vector of values. If the buyer enters a trade, the *decUnits* method reduces its number of units by one. The *setUnits* method sets the number of units. The *getUnits* method retrieves the number of units the buyer has left.

The *getBid* method formulates a bid given a global counter and the running average price. At the beginning of the trading period, the buyer formulates an offer by picking a bid between the minimum bid and its true value for the unit up for trade. As the number of offers increases and the running average price is known, the bidding strategy changes. The bid is set equal to the weighted sum between a willingness to pay converging to its true value and the running average price.

Market-maker

Like the other classes, upon being instantiated the market-maker gets its type, seed, and index. Unlike the buyer and seller, the market-maker does not get a vector of induced valuations or number of units for each trading period. Instead, it gets an initial set of dynamically updatable parameters to formulate its offers throughout the trading periods.

Market-makers are endowed with budgets and a set of units for trade. A static maximum value and a minimum cost keep its offers within reasonable bounds. It gets an inventory cost, interest rate, and an arbitrarily small number (δ). It also gets an array of forecasted equilibrium prices of an arbitrary number of future periods.

At any point in a trading period, the market-maker can enter an offer to buy or sell a unit. Like the buyer, it has a *getBid* method to formulate a bid. Like the seller, it has a *getAsk* method to formulate an ask.

Unlike both the buyer and the seller, the market-maker does not use a period's sequence counter or the running average price to formulate an offer strategy. It uses the theoretically predicted equilibrium (clearing) prices from the microsimulation to generate an estimated expected value over all clearing prices and the array of forecasted clearing prices. It calculates a discounted value and a discount cost using these parameters along with inventory cost and interest rates.

Also unlike the buyer and seller, the number of offers in a period does not affect a market-maker's offer strategy. When entering a bid, if its discounted value is greater than the standing ask by a small amount it sets the bid equal to the standing ask. Otherwise, it formulates a random normal bid uniformly distributed between the standing ask and the standing ask plus a small number. When entering an ask, if the discounted cost is less than standing bid by a small amount the market-maker sets the ask equal to the to the standing bid. Otherwise, it formulates a random normal ask uniformly distributed between the standing bid and the standing bid plus a small number.

Finally, the market-maker has one method to set and one method to retrieve every attribute value. It is a very extensive list and thus forgone in this documentation.

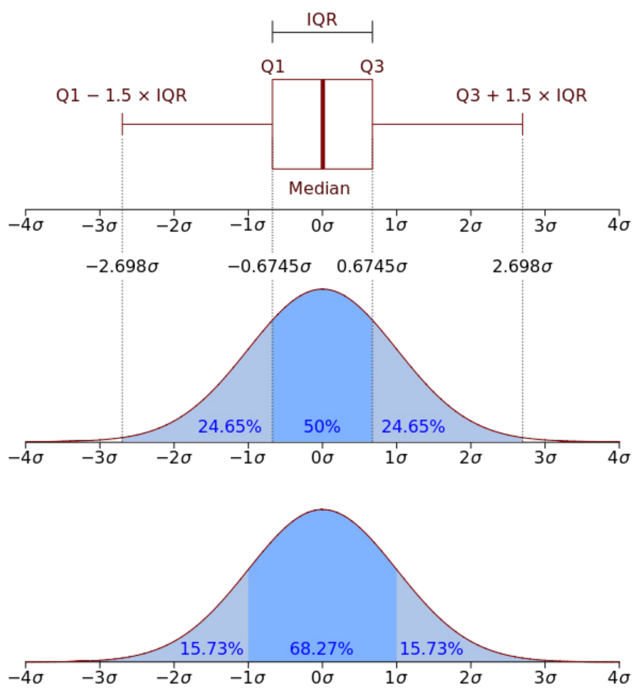
Statistical Analysis Module

The *statistical analysis module* consists of *exploratory data analysis* (EDA) and *descriptive statistics*. Box and scatter plots provide graphical representations of the main characteristics each the entire dataset. Mainly, I use non-parametric analysis to examine the central tendency, variability and dispersion of each model version's dataset of prices and gains from trade.

By comparing the central tendency and dispersion of each period's set of market clearing prices to determine price volatility. Comparing the each period's distribution of trade contract prices with the theoretical intratemporal and intertemporal equilibrium prices gives us the goodness-of-fit of simulated outcomes. The same process comparing simulated gains from trade and theoretical economic surpluses allows to examine the intratemporal and intertemporal market efficiencies. Finally, comparing the change in simulated market-clearing prices across periods with the change in theoretical predicted values shows us the level of intertemporal price variability.

This statistical analysis program is written in STATA, which is a general-purpose statistical analysis software package.

Box-plot analysis is used to have visually discernible measurements of mean and standard deviation. Also the box-plot allows the information to be parsed into the 25^{th}, 50^{th} and 75^{th} percentiles. This are measurements of the bottom quartile, median and top quartile. And they are represented by the bottom box, the band near the middle and the top box, respectively. The top and bottom outliers, or whiskers, represent the the largest and smallest observations. They are joined by the outer lines that represent one standard deviation from the mean.



The box-plots are divided into two groups. The first group represents the baseline results, which is the outcome of the *agent-based* simulation without market-makers. The second group represents the results with market-makers in the *agent-based* simulation.

This module analyzes the mean and standard deviation of the trading prices and market clearing prices of both groups. It also analyzes the mean and standard deviation of units traded for both groups. These measurements are compared with the theoretical predictions of the *microsimulation* data to assess their fitness.

By comparing variations in prices and units traded between groups, this module shows the effects of the market-makers in intertemporal price volatility and intertemporal market efficiency.